Part III

Slope

Slope Stability Analysis

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Slope Stability Analysis

Stability analyses of slopes created by both embankments and excavation are the subjects most frequently encountered in geotechnical engineering. A slope naturally is subjected to potential energy due to its self weight. If pore water pressure, surcharge, seismic loads and wave forces are exerted on a slope, slope instability may arise. When the shear stress due to self weight and external loads exceeds the shear strength of the soil, slope failure will occur. Determining such shear stress and soil strength pertains to slope stability analysis.

Slope stability has been conventionally analyzed by performing displacement and stability analyses separately (Duncan, 1984; Huang, 1983; Brunsden et al, 1984). However, actual slope failure is caused by large displacements from gradual increase in deformations in local areas. The displacements and the process of failure are not independent, but they take place in the process of progressive failure (Chowdhury, 1978; Griffiths, 1993). Therefore, it is important to formulate a slope stability analysis method, which can continually track the failure process from the initial deformation to ultimate failure.

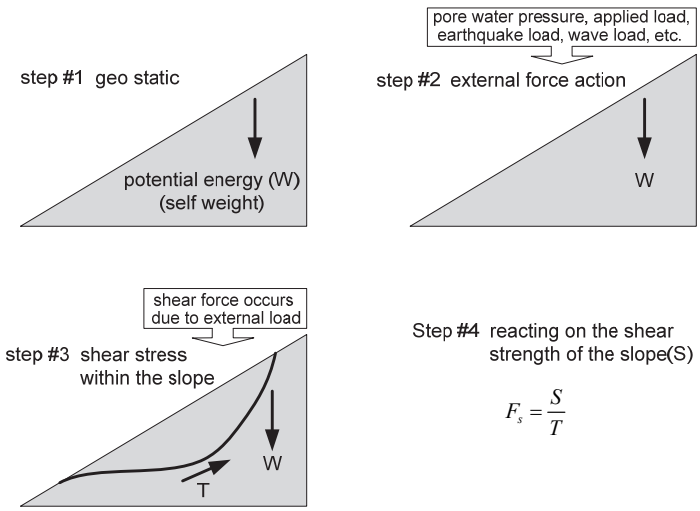


Figure 1 Slope Failure Schematic Diagram

The following methods are suggested for slope stability analysis:

- Limit equilibrium theory by Mass procedure and Slice method
- Limit theory
- Finite element method by elastoplastic theory

The limit equilibrium theory has been most widely used for slope stability due to its simplicity and numerous application cases, but it retains the drawback in calculating only the minimum safety factor. In order to monitor the stability of a slope on site, its failure behavior must be analyzed in addition to calculating the minimum safety factor. For managing progressive slope failure, the locations and the control values for monitoring must be determined in advance. The finite element method provides suitable analysis results and is also used to determine the progressive failure behavior of a slope (Anderson et al, 1987; Duncan, 1996).

With the advent of on-going development in computational technology, the finite element method has been widely used for analyzing ground behavior for displacements, stresses, pore water pressure, etc. The use of the finite element method has been recently extended to evaluating the stability against slope failure from simple behavioral analysis for ground structures.

SoilWorks basically provides the limit equilibrium method as well as the strength reduction method, which solves slope stability problems by the finite element method, and the stress analysis method, which is based on the limit equilibrium method.

Table 1 Comparison between limit equilibrium method and finite element method

Slope analysis	Limit equilibrium method	Finite element method
Analysis principle	- Slice method - Equilibrium of forces and moments applied	- Finite element modeling - Constitutive equations
Analysis results	- Safety factor, critical section	- Deformed shape, stress distribution
Pros	- Simple theory - Numerous successful construction cases	- Various ground properties analysis - Displacement, stress results
Cons	- Unable to assess deformed shape and stress distribution at failure - Assumptions required	- Difficult to analyze results - Lack of stability evaluation data
Input data	- Cohesion, internal friction angle	- Input data for constitutive models (Mohr Coulomb in SoilWorks)

Table 2 Elements, loads & boundaries used in slope stability

		SRM	SAM	LEM
Ground elements	Plane strain	√	√	X
	Geometry	X	X	√
Structural elements	Beam	√	√	X
	Truss / Embedded	√	√	X
	Pile & Reinforcement	√	√	X
	Interface	√	√	X
	Spring	√	√	X
	Elastic Link	√	√	X
	Rigid Link	√	√	X
	Geogrid	√	√	X
	Spring	√	√	X
	Reinforcement (LEM)	X	X	√
Loads	Loads	√	√	√
	Loads (tunnel module)	√	√	X
	Line loads (LEM)	X	X	√
	Static seismic loads (LEM)	X	X	√
Boundaries	Boundaries (tunnel module)	√	√	X
	Arc Failure Surface	X	X	√
	Polygonal Failure Surface	X	X	√
	Auto-searched Surface	X	X	√
	Cut Failure Surface	X	X	√
	Arc Passing Limit	X	X	√
	Tension Crack	X	X	√
Ground properties	Elastic material	Δ	√	X
	Mohr Coulomb	√	√	X
	Mohr Coulomb (LEM)	X	X	√
Structural properties	Elastic material	√	√	X

where, √ denotes “can be used”, X denotes “cannot be used” and △ denotes “can be used with limitation”. If elastic ground is used in the strength reduction method, the results are meaningless retaining always the maximum safety factor defined by the user.

Chapter 1

Strength Reduction Method

1.1 Introduction

The finite element numerical method is a precise analysis method, which satisfies the force equilibrium, compatibility condition, constitutive equation and boundary condition at each point of a slope. It simulates the actual slope failure mechanism and determines both the minimum factor of safety and the failure behavior. It can also reflect real in-situ conditions better than any other methods. Moreover, it can simulate the failure process without having to assume any failure surfaces in advance. (Griffith et al 1999; Matsui, 1990).

Two types of methods exist in the finite element method in analyzing slope stability, which are the direct method using the strength reduction method and the indirect method of analyzing the safety factor in combination with the limit equilibrium method (Pasternack, S.C. and Gao, S, 1988). In the strength reduction method, the shear strength (c, ϕ) of the sloped ground material is gradually reduced until the point of divergence in calculation at which point slope failure is assumed to have taken place. The maximum strength reduction ratio at that point is considered to be the minimum safety factor.

Such methods require iterative nonlinear analyses consuming significant analysis time, but they are manageable through the advancement in computing speed.

The advantages of using the strength method are summarized below.

1. Failure surfaces need not be assumed. Failure takes place when the shear strength of the soil is less than the shear stress due to the self weight of the soil. Details are further outlined in the subsequent section.
2. The finite element method does not require the data or concept of slices and satisfies the equilibrium state until failure.
3. Stresses and deformations of the in-situ slope can be obtained through analysis.

1.2 Theoretical Background

The strength reduction method using the finite element technique was first proposed by Zienkiewicz (1975). A Gauss point, A, of an element in a sloped ground structure is now focused on to calculate the factor of safety of the slope as shown in Figure (1.2.1). The stress state at this point is represented in a Mohr circle. In order to simulate the sliding surface, the shear strength at the point is divided by a factor of safety, F, so that the Mohr circle for the stress state of the fictitious sliding surface becomes tangent to the failure envelope. That is, the stress state of the point is corrected to the failure state. An increase in the number of such failure points results in a global slope failure. At that point in the finite element analysis, the calculation diverges, the analysis stops, and the limit value, F, becomes the minimum factor of safety for the slope. This method requires stability in numerical analysis in order to generate consistent results and evaluate the actual failure behavior.

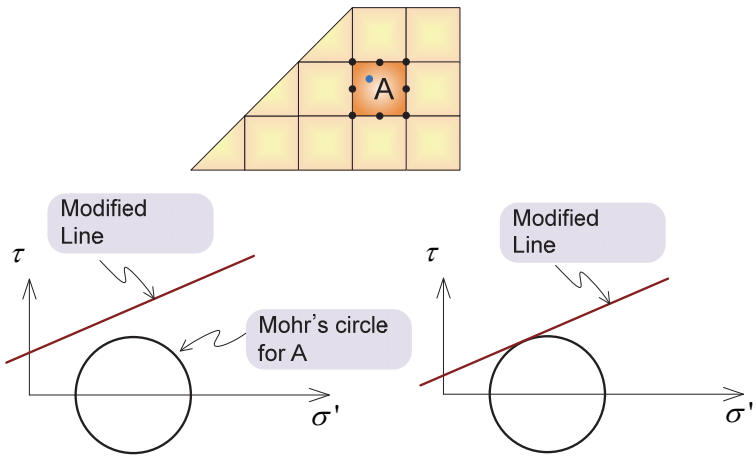


Figure 1.2.1 Strength Reduction Method

1.3 Method of calculating Minimum Safety Factor

The ground model of Mohr Coulomb is used for the strength reduction method for which the following input parameters are used. The modulus of elasticity (E) and Poisson's ratio (ν) are assumed to be constant. The cohesion (c), friction angle (ϕ) and dilatancy angle (ψ) are gradually reduced, and the factor of safety, F_s is determined at the diverging point in calculation. The factor of safety against slope failure is determined on the basis of shear failure as follows:

$$F_s = \frac{\tau}{\tau_f} \tag{1.3.1}$$

where,

τ : Shear strength of slope material, which is calculated on the basis of the Mohr-Coulomb criterion.

$$\tau = c + \sigma_n \tan\phi \tag{1.3.2}$$

The shear stress at the sliding surface, τ_f is expressed in Eq. 1.3.3.

$$\tau_f = c_f + \sigma_n \tan\phi_f \tag{1.3.3}$$

where,

$c_f = \frac{c}{\mathrm{SRF}}$: Coefficient of shear strength

$\phi_f = \tan^{-1}\left(\frac{\tan\phi}{\mathrm{SRF}}\right)$: Coefficient of shear strength

SRF : Strength reduction factor

In the strength reduction method, the value of SRF is evaluated to be the factor of safety, which is dependent on the number of iterations for convergence and the unequilibrated force norm specified by the user. SoilWorks provides the default values of 50 iterations and the norm of 0.03. In order to accurately determine the minimum factor of safety, F_s must be increased in a very small increment even though it may extend the analysis time.

The user needs to pay attention to the sensitivity of the dilatancy angle to obtain a realistic deformed shape and the factor of safety. When the dilatancy angle (ψ) is equal to the friction angle (ϕ), the stiffness matrix becomes symmetrical resulting in the advantage of fast analysis and convergence, while being in the associated plastic condition. But the ground tends to swell excessively. In case ψ =0, the site condition can be realistically simulated and the factor of safety becomes more credible because no change in volume exists, while being in the non-associated plastic condition.

Chapter 2

Stress Analysis Method

2.1 Introduction

Slope stability analysis is largely divided into simplified methods and numerical analysis methods. The limit equilibrium method being a simplified method is most widely used in solving slope stability problems in practice. However, this method cannot consider the effect of stress history during the process of slope formation and changes in ground stress due to groundwater. On the contrary, the finite element method, being a widely used numerical method, can consider the process of slope formation and other ground properties, but requires extensive analysis time and lacks slope safety evaluation criteria. Among many researches combining the advantages of the simplified and numerical methods, SoilWorks uses the finite element method based on the work of Kim (1998) in KAIST for slope stability analysis.

This method first performs stress analysis for a slope using the finite element method. Then the stress analysis results are used to calculate the factors of safety for a number of potential sliding surfaces assumed for the limit equilibrium method. Among the cases, the minimum factor of safety and the corresponding critical section is determined.

The constitutive model of the ground material in finite element analysis uses the Mohr-Coulomb yield criterion, which is identical to that of the limit equilibrium method.

2.2 Evaluation of Safety Factor

The safety factor used in the finite element method is defined as,

$$F_s = \frac{\int_S \tau_f d\Gamma}{\int_S \tau_m d\Gamma} \tag{2.2.1}$$

τ_m is the induced shear stress, and τ_f is the shear strength according to the Mohr-Coulomb failure criterion.

$$\begin{aligned} \tau_f &= c + \sigma_n \tan\phi \\ \tau_m &= \frac{1}{2}\left(\sigma_y - \sigma_x\right)\sin 2\theta + \tau_{xy}\cos 2\theta \end{aligned} \tag{2.2.2}$$

where, the stress, σ_n, normal to the sliding surface is given as,

$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - \tau_{xy}\sin 2\theta \tag{2.2.3}$$

where,

c	: Cohesion
ϕ	: Internal friction angle of material
θ	: Angle between the horizontal surface and sliding surface
σ_x and σ_y	: Normal stresses in the directions of x and y respectively
τ_{xy}	: Shear stress

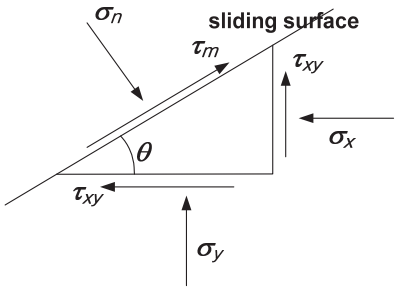


Figure 2.2.1 Stress components of sliding surface

In order to calculate the safety factor defined in Eq. (2.2.1), stress integration is required along the potential sliding surface. Since the stress integration results of the continuous stress fields defined at the nodes are more reliable than those at the Gaussian integration points, Soilworks calculates the continuous stress field at each node using the global stress smoothing method (Hinton & Compbell, 1974).

2.3 Stress Integration along Potential Sliding Surface

Integration of shear stresses along the potential sliding surface is required to evaluate the factor of safety as defined in Eq. (2.2.1). The integration is performed on all the elements along the path of the sliding surface.

The shear stress at a point within an element is calculated from the nodal stress calculated by the global stress smoothing method.

$$\boldsymbol{\sigma} = \sum_{i=1}^{node} \mathbf{N}_i \, \boldsymbol{\sigma}_i^{node} \tag{2.3.1}$$

where,

N_i : Shape function at node i

σ_i^{node} : Nodal stress at node i

σ : Stress at a point within an element.

The stress integration over the potential sliding surface in the two-dimensional global coordinate system is performed after transforming the stress into the integral form in a one-dimensional local coordinate system.

$$\int_{n1}^{n2} \tau(x,y)\, d\Gamma = \frac{L}{2}\mathbf{T}\int_{-1}^{1} \tau(\xi)\, d\xi = \frac{L}{2}\mathbf{T}\sum_{i=1}^{\mathrm{n_{int}}} W_i \tau(\xi_i) \tag{2.3.2}$$

where,

ξ : Coordinate variable in local coordinate system

W_i : Integral constant at the integration point i

$\mathbf{T}$: Transformation matrix transforming the stress from a local coordinate system into the global coordinate system

L : Element length

τ : Shear stress (τ_m) or shear strength (τ_f) on the potential sliding surface

The global safety factor for the final potential sliding surface is given as,

$$F_s = \frac{\sum_{i=1}^{nel} \int_{n_1}^{n_2} \tau_f d\Gamma}{\sum_{i=1}^{nel} \int_{n_1}^{n_2} \tau_m d\Gamma} \tag{2.3.3}$$

where,

nel	: Number of elements intersecting along the potential sliding surface
n_1	: Starting point of the potential sliding surface within an element
n_2	: Ending point of the potential sliding surface within an element

The stress analysis method uses the stress field obtained on the basis of the finite element analysis method and the failure surface used in the limit equilibrium method. SoilWorks as such is optimized for using the advantages of attaining the stress distribution and deformed shape from finite element analysis and the critical section from the limit equilibrium method. This approach significantly reduces the analysis time and accommodates various ground and reinforcement materials without having to make any special assumptions.

Chapter 3

Limit Equilibrium Method

3.1 Introduction

The limit equilibrium method uses equilibrium of forces and moments of a sliding surface relative to the total ground mass. Based on this concept, its basic assumption is to satisfy the Mohr-Coulomb failure criterion along the assumed surface of straight lines and arcs or an irregular surface. At the moment of failure of the sliding surface, the stability of the ground mass is calculated. This method uses a simple principle of statics to obtain the solution. The limit equilibrium method is widely used in the slope stability of clay, which is well known for its usability and credibility through accumulated experience. Among many slope stability analysis methods based on the limit equilibrium theory, the accuracy of the solution depends on the accuracy of the strength parameters and geometric condition of the slope and the inherent accuracy of each analysis method.

The analysis methods include $\phi = 0$, Fellenius, Bishop, Modified Bishop, Janbu, Spencer, Morgenstern and Price, general limit equilibrium (GLE), soil wedge analysis and logarithmic spiral analysis methods.

Slope stability analysis using the limit equilibrium method requires appropriate assumptions to overcome the indeterminate problem of the number of unknowns being greater than the number of equations. If the number of slices in Figure 3.1.1 is n, the number of equations becomes $3n$, which is consisted of equilibrium equations for vertical and horizontal forces and moments.

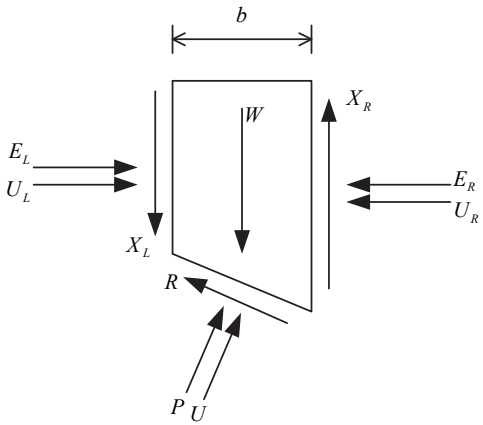


Figure 3.1.1 Forces acting on a slice

3.2 Analysis Methods

3.2.1 Fellenius Method

The Fellenius method is known as the ordinary method of slices or the Swedish method. The factor of safety of a slope is calculated on the basis of equilibrium of moments at the center of the sliding surface. The sum of acting forces between individually divided slices is assumed to be in equilibrium. The resisting force against sliding is defined by the shear resisting capacity, which is obtained by the shear stress multiplied by the base area of the slice.

$$R = \tau l \tag{3.2.1}$$

The shear strength, τ is calculated by the Coulomb's failure criterion.

$$\tau = c' + (\sigma - u)\tan\phi' \tag{3.2.2}$$

where,

c'	: Cohesion based on effective stress
σ	: Total stress at the base of a slice
u	: Pore water pressure
ϕ'	: Angle of internal friction based on effective stress

The shear force, T , which induces sliding of a slope, is obtained by,

$$T = W\cos\alpha \tag{3.2.3}$$

where,

T	: Sliding force acting at the base of a slice
W	: Total weight of a slice
α	: Slope of sliding surface of a slice

Finally the factor of safety by the Fellenius method is given by,

$$F_{Fellenius} = \frac{\Sigma\left[c' + (\sigma - u)\tan\phi'\right]l}{\Sigma W\cos\alpha + \Sigma M_{ext}/R} \tag{3.2.4}$$

3.2.2 Simplified Bishop Method

The simplified Bishop method is most widely used among various slice methods due to its simplicity and relative accuracy.

Bishop proposed that the sum of forces acting on the vertical faces of a slice would be assumed to act in the horizontal direction. This method uses the center of virtual rotation, which can be also applied to a non-arc sliding surface as follows:

$$X_R - X_L = 0 \tag{3.2.5}$$

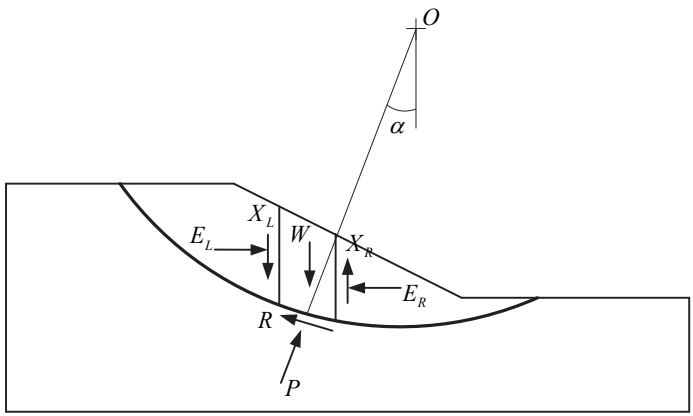


Figure 3.2.1 Slice forces by Bishop

The equilibrium equations at the sliding surface are as follows:

$$\begin{aligned} R &= \frac{1}{F}\left\{c'l + (P - ul)\tan\phi'\right\} \\ P\cos\alpha + R\sin\sigma &= W - (X_R - X_L) \end{aligned} \tag{3.2.6}$$

Since $X_R - X_L = 0$ is assumed, P can be defined as,

$$P = \left\{ W - \frac{1}{F}\left(c'l\sin\alpha - ul\tan\phi'\sin\alpha \right) \right\} \frac{1}{m_\alpha} \tag{3.2.7}$$

where, $m_\alpha = \cos\alpha \left(1 + \frac{\tan\alpha\tan\phi}{F} \right)$

Accordingly, considering the equilibrium of moments about the center of the arc, O as shown in Figure 3.2.1, the factor of safety, F can be expressed as,

$$F = \frac{1}{\sum W\sin\alpha} \sum \left[\left\{ c'l\cos\alpha + (W - ul\cos\alpha)\tan\phi' \right\} \left\{ \frac{1}{m_\alpha} \right\} \right] \tag{3.2.8}$$

The simplified Bishop method retains the safety factor term, F, on the left and right hand sides. As such, the final F needs to be calculated by a try and error method through an iterative process.

In the process of a try and error method, SoilWorks uses the Fellenius' safety factor as an initial value of F. This prevents unstable convergence and promotes fast analysis through a small number of iterations.

3.2.3 Simplified Janbu Method

The simplified Janbu method is very similar to that of the Bishop's method except that the Janbu's method uses the equilibrium state of horizontal forces rather than the moment equilibrium used in the Bishop's method.

$$X = 0 \tag{3.2.9}$$

Accordingly, good results can be expected through this method when a slope is consisted of a number of layers of different materials.

The equilibrium equation at the sliding surface is identical to that of the simplified Bishop

method (ref. Eq. (3.2.6). Therefore, P can be rearranged as,

$$P = \left\{ W + \left(X_R - X_L \right) - \frac{1}{F} \left(c'l \sin\alpha - ul \tan\phi' \sin\alpha \right) \right\} \frac{1}{m_\alpha} \tag{3.2.10}$$

where, $m_\alpha = \cos\alpha \left(1 + \frac{\tan\alpha \tan\phi}{F} \right)$

Considering the equilibrium state of horizontal forces ($X = 0$), the factor of safety, F , can be expressed as,

$$F = \frac{\Sigma \left\{ c'l + \left(p - ul \right) \tan\phi \right\} \cos\alpha}{\Sigma p \sin\alpha} \tag{3.2.11}$$

The Janbu's method, like the Bishop's method, the final safety factor, F , is obtained through an iterative analysis process.

3.2.4 Spencer Method

The Spencer method takes into account the equilibrium conditions of the horizontal force and equilibrium conditions of the moment. It assumes that there is a linearly propotional relationship between with the horizontal force and the normal force of slice which is considered to zero in the simplified method of Yanbu and the simplified method of Bishop.

$$X / E = \tan\theta \quad \text{or} \quad \lambda \tag{3.2.12}$$

Here, θ : Angle between horizontal force and normal force, i.e., the direction of the resultant of the forces acting on the slice

λ : Ratio of horizontal force and normal force

By entering the normal force(Eq. 3.2.10) into both the force equilibrium equation (Eq. 3.2.11) and the moment equality equation (Eq. 3.2.8), the full equilibrium will be iteratively calculated until it finds λ that makes two safety factors equal.

The method of Spencer, because it is a way to satisfy all the conditions of equilibrium of

forces and moment balance, can advantageously provide more accurate safety factor. However, in order to consider both equilibrium of force and moment, it may also occur if the convergence doesn't reach relatively compared to the simplified method of Yanbu and the simplified method of Bishop, and its analysis time is longer.

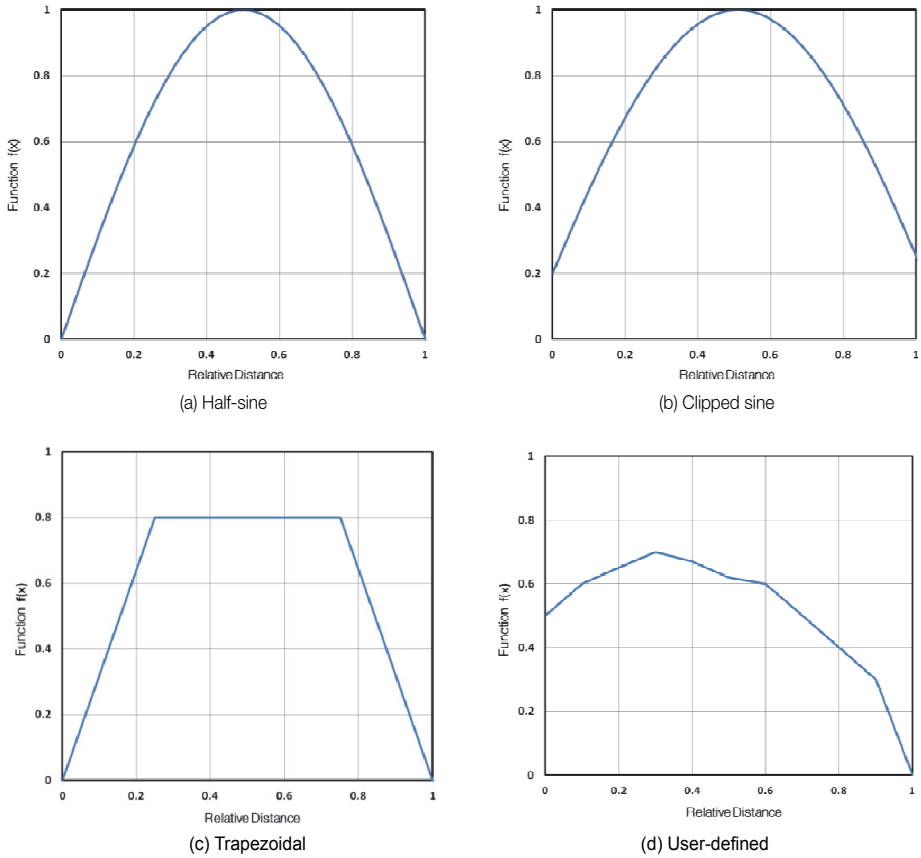
3.2.5 Morgenstern-Price Method

The Morgenstern-Price method is a method that takes into account both equilibrium of the force and the moment in the same manner as the Spencer method. However, unlike The Spencer method which assumed the normal and horizontal forces of the slice to have a linearly proportional relationship, the Morgenstern-Price method assumed the relationship in the form of a particular function.

$$X / E = \lambda f(x) \tag{3.2.13}$$

Here, $f(x)$: Normal force distribution function

The normal force distribution function can be defined using half-sine, clipped sine, trapezoidal, and User-defined as shown in Figure 3.2.2. The Morgenstern-Price method has the advantage of predicting the distribution shape of the normal force since it assumes and analyzes various normal force distribution functions.



(a) Half-sine

(b) Clipped sine

(c) Trapezoidal

(d) User-defined

Figure 3.2.2 Normal force distribution function of the Morgenstern-Price method

3.2.6 Sarma Method (Vertical)

The Sarma method considers both force and moment equilibrium of the various direction of slice including the vertical direction. SoilWorks currently applies the Sarma method in the vertical direction slice only.

The Sarma method presented the relationship of horizontal force and the vertical force acting on the slice by the Coulomb failure criterion as compared to a simple function relationship assumption in the Morgenstern-Price method and Spencer method.

$$X = \lambda \times \left(Ch + E \tan \phi' \right) \tag{3.2.14}$$

Here, C : the cohesive strength of the slope

h : Height of the slice

ϕ' : Friction angle of slope

Theoretically the Sarma method is easy to understand because it applies the failure criterion question rather than a specific function for the relationship between the normal and horizontal forces of the slice. However, it remains difficult to assume the cohesive strength of the slope and the friction angle. If the cohesive strength is small, the result of the Sarma method becomes similar to the Morgenstern-Price method and the Spencer method. Therefore, the method is applied mainly to the analysis of small rock slope or a place where cohesion is not expected.

3.2.7 Comparison of analysis methods

Table 3.2.2 compares the characteristics of each analysis method. In the simplified method of Janbu and the simplified method of Bishop, by simplifying the equilibrium condition, the analyses are relatively simple and have the advantage of convergence in iterative calculation. However, since the equilibrium condition is omitted, they generate inaccurate result in some cases.

On the other hand, the Spencer method, the Morgenstern-Price method, and the Sarma method considers the force and moment equilibrium, their analysis is time-taking and it causes problems in convergence in some cases. However, due to the equilibrium of force

and moment, they deliver more accurate result.

Table 3.2.1 Types of LEM analysis methods provided in SoilWorks

Analysis Method	Assumption	Vertical Equilibrium	Horizontal Equilibrium	Moment Equilibrium
Simplified Bishop	$X_R - X_L = 0$	Yes	No	Yes
Simplified Janbu	$X = 0$	Yes	Yes	No
Spencer	$X / E = \tan\theta$ or λ	Yes	Yes	Yes
Morgenstern-Price	$X / E = \lambda f(x)$	Yes	Yes	Yes
Sarma (vertical)	$X = \lambda \times (Ch + E \tan\phi')$	Yes	Yes	Yes

Table 3.2.2. Comparison of LEM analysis methods provided in SoilWorks

Analysis Method	Strength	Weakness
Simplified Bishop	- Short analysis time - Applicable to arc / Polygonal	- Inaccurate result when horizontal force is acting (i.e., seismic load)
Simplified Janbu	- Short analysis time - Suitable for shallow slope	- More conservative result
Spencer	- Applicable to arc / Polygonal - More accurate safety factor	- Longer analysis time - More sensitive convergence
Morgenstern -Price	- Predictable internal normal force - More accurate safety factor	- Longer analysis time - More sensitive convergence
Sarma (vertical)	- Suitable for rock slope analysis - More accurate safety factor	- Longer analysis time - More sensitive convergence - Assumption needed for cohesive strength and friction angle

3.3 Definition of Slope Failure Surface

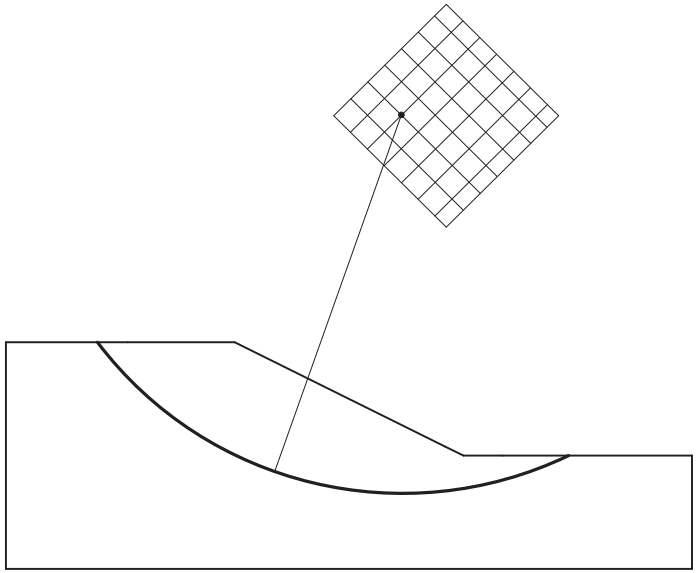
Arc failure surfaces and multi-linear failure surfaces can be defined in the limit equilibrium method. In case of arc failure surfaces, a zone of arc centers must be defined first rather than defining the failure surfaces themselves. In order for the user to readily define failure surfaces, SoilWorks supports various ways of supplementary settings.

Engineering judgment by extensive experience is required in defining the zone of arc centers however. When such judgment is not so obvious, arc failure surfaces can be defined by the function of automatic definition.

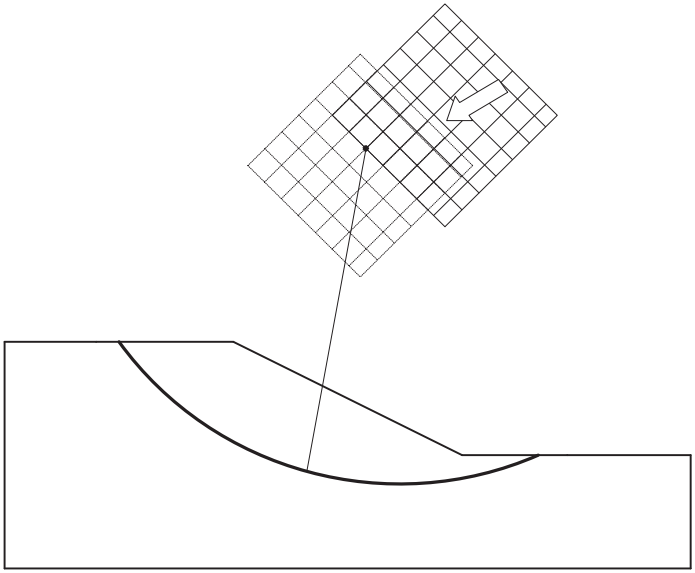
In case of multi-linear failure surfaces, tabular input or mouse clicks are supported to readily define the surfaces.

3.3.1 Arc Failure Surface

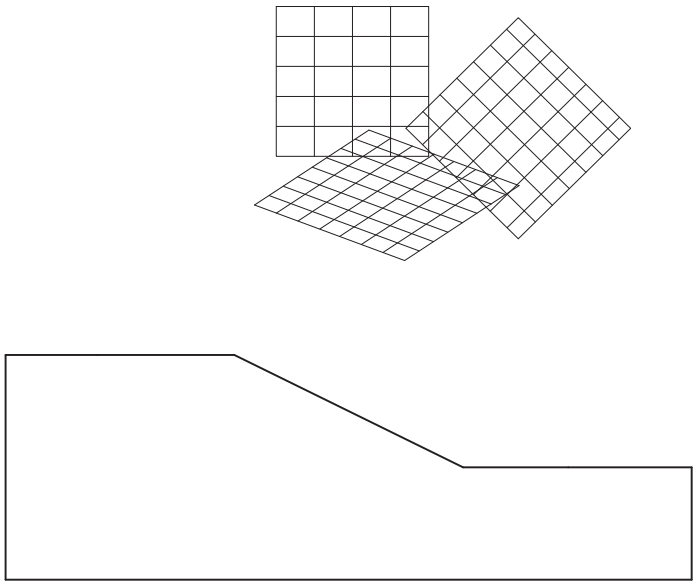
Prior to defining the failure surfaces, the zone of arc centers is defined first at a location where the minimum factor of safety is anticipated as shown in Figure 3.3.1(a). If the arc center corresponding to the minimum safety factor is found to be at the periphery of the zone after the analysis as shown in Figure 3.3.1(b), the zone of arc centers must be relocated for re-analysis so that the arc center corresponding to the minimum safety factor falls within the middle of the arc center zone. In addition, SoilWorks allows the user to define a number of zones of arc centers as shown in Figure 3.3.1(c) and analyze the multiple cases in a single analysis run such that the arc surface corresponding to the minimum safety factor can be found much more quickly.



(a) Selection of a zone of arc centers



(b) Relocation of the zone of arc centers for reanalysis in case the arc associated with the minimum safety factor is located at the periphery of the zone



(c) Selection of multiple zones of arc centers

Figure 3.3.1 Selection of zones of arc centers

When defining arc failure surfaces by individual arc centers included in the specified zone, the arc tangent method and the arc-radius-length method can be used.

3.3.1.1 Arc Tangent Method

The arc tangent method is based on defining a series of straight lines to which the arcs drawn from the arc centers become tangent as shown in Figure 3.3.2. The zone of arc centers can be simply specified by a mouse click for this operation. From each arc center, a number of arcs and a radius to generate failure surfaces can be defined by specifying the "number of incremental arc radii", "arc radius to be increased" and "change tangent direction.

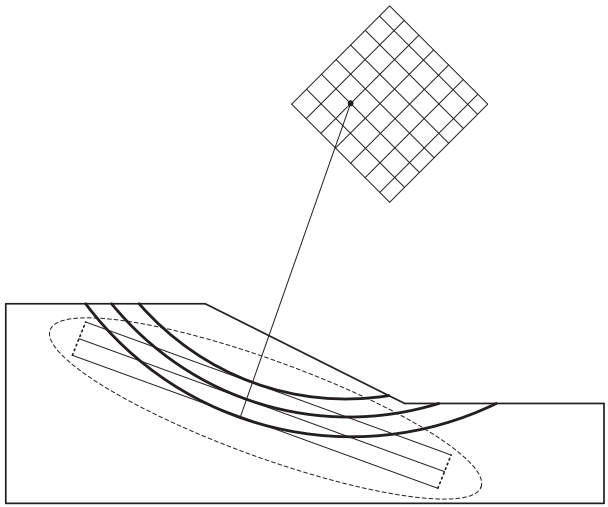


Figure 3.3.2 Definition of failure surface arcs by arc tangent method

3.3.1.2 Arc Radius-Length Method

In the method of using arc radius and length, the “shortest arc radius method” and “defining a through point” functions exist. Limitations can be also imposed to the arcs defined by both functions through “layer set limiting passage”. Each function is explained below.

a. Shortest Arc Radius

The function of “Shortest Arc Radius” searches a node and a straight line, which are located at the shortest distances from an arc center to the slope. The shorter of the two distances is defined as the shortest arc radius, $r_{\min}$. Using the shortest radius and a defined increment, Δr , n times, n radii can be defined as follows:

$$r_i = r_{\min} + \Delta r \times i \qquad (i = 1 \cdots n) \tag{3.3.1}$$

where,

r_i : i th radius

$r_{\min}$: Shortest radius found from search

Δr : Specifies radius increment

n : Number of radius increments

b. Defining a Through Point

SoilWorks finds the distance from an arc center to a through point, A as shown in Figure 3.3.3 and defines it as the initial arc radius value, r_0. The initial radius is increased by an increment, Δr, n times resulting in n arc radii defined as,

$$r_i = r_0 + \Delta r \times (i-1) \qquad (i = 1 \cdots n) \tag{3.3.2}$$

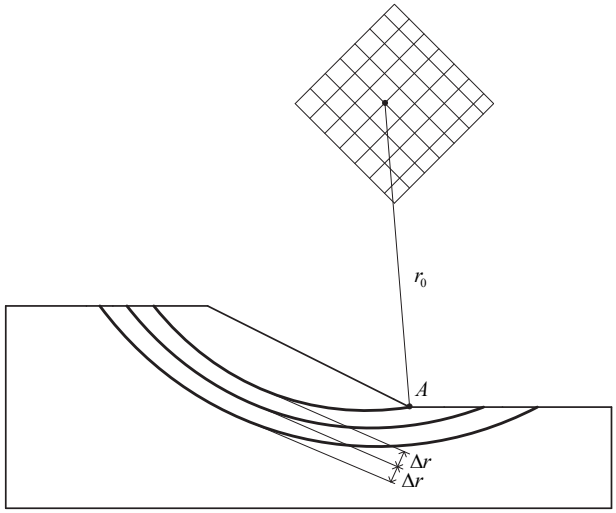


Figure 3.3.3 Definition of arc radii by a through point

c. Layer Set Limiting Passage

Layer Set Limiting Passage is a supplementary function for the above two arc definition functions, which limits the generated arcs from passing through a specific layer. In the example of Figure 3.3.4, three radii have been generated while the limitation to pass

through the layer A has been imposed. The third arc is then ignored and adjusted to connect to the point C, which forms the shortest radius from the arc center in question to the layer A. The number of radii defined from the corresponding arc center in this example still remains at 3.

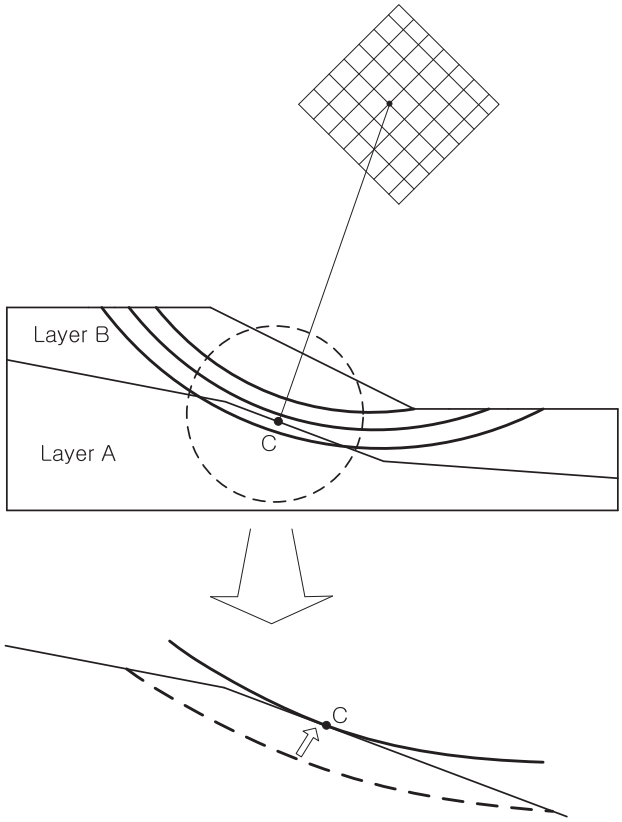


Figure 3.3.4 Definition of radii when layer set limiting passage is specified

3.3.2 Definition of Automatic Arc Failure Surfaces

The desired analysis results and analysis time are largely dependent on defining the zone of arc centers. But if the selection of the zone is not so obvious, SoilWorks can help the user automatically search and define the zone of arc centers. Auto-definition of arc failure

surfaces to obtain the arc center associated with the minimum safety factor can be performed by specifying a few parameters, the number of horizontal spaces, arc radius increment, number of arc incremental radii, through point and limit point.

The zone between the through point and the limit point is divided by the number of horizontal spaces (shown as 5 - ΔX in Figure 3.3.5). As many as the horizontal spaces, straight lines are constructed, which connect the through point and the intersections of the ground surface and the vertical lines (dotted in Figure 3.3.5) of the horizontal spaces. These lines are denoted as L1 through L5 in the ground in Figure 3.3.5. The same number of line A's is then constructed in the perpendicular directions to the lines L1 through L5 at the mid points. The arc centers are now defined along the line A's by the arc radius increments and the number of increments. The arc radius increments are determined by the lines L1 through L5. The first two increments of each line A are reduced to one half. The total number of the arc centers then becomes as follows:

Total number of arc centers = No. of spaces (line A's) X No. of arc radius increments

SoilWorks automatically undergoes two phases of defining the arc failure surfaces. Analysis for arc failure surfaces using the above arc centers is first performed to find the location of the arc center producing the minimum factor of safety. The second phase of analysis involves the investigation of arc failure surfaces using the arc centers further refined around the arc center corresponding to the minimum safety factor found in the first phase.

Supposing the arc center associated with the minimum safety factor found in the first phase to be the point A in Figure 3.3.6, which lies on the line B1, the adjacent lines B2 and B3 are found to be the boundary of the new set of arc centers. If the point A exists at the i-th arc center on the line B1, the arc centers at the i-1th and i+1th arc centers on the lines B2 and B3 are located, denoted as A1, A2, A3 & A4 in Figure 3.3.6. The zone bounded by the 4 points is divided by the number of spaces specified by the user to define a new set of arc centers for the second phase of analysis.

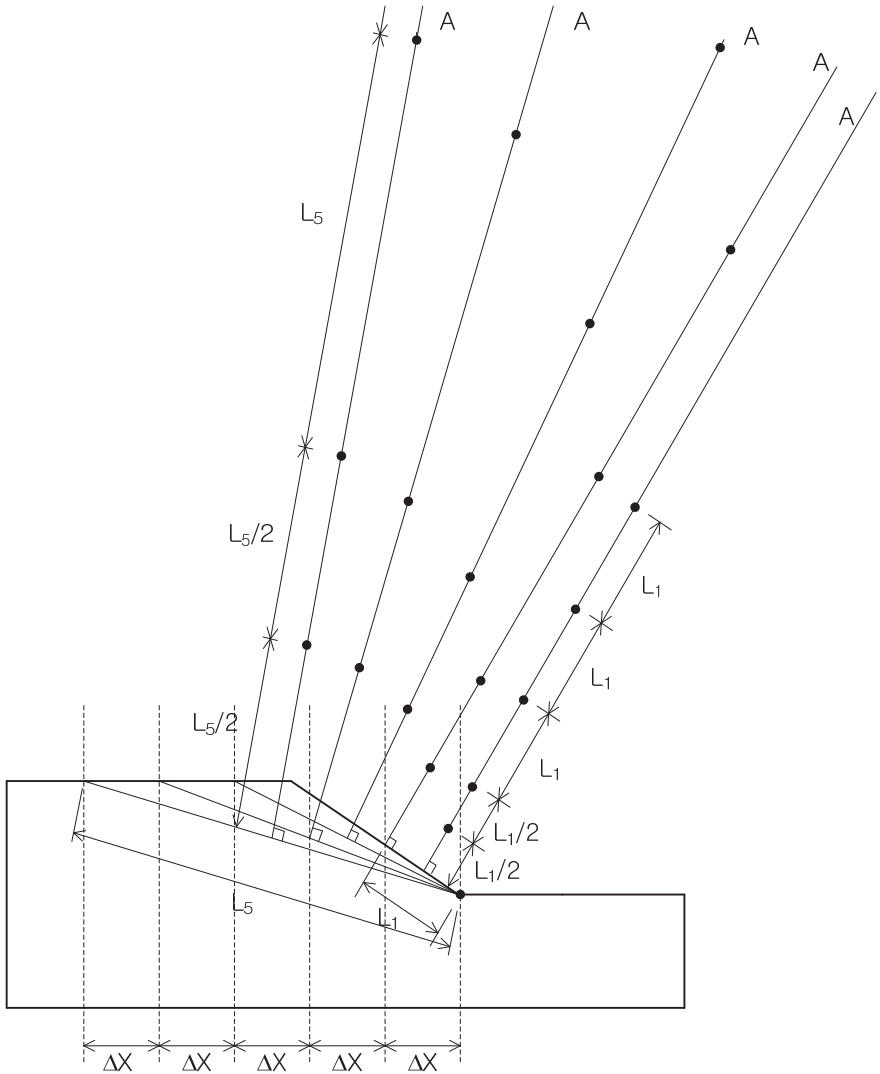


Figure 3.3.5 Auto-defining arc centers for the 1st phase of analysis

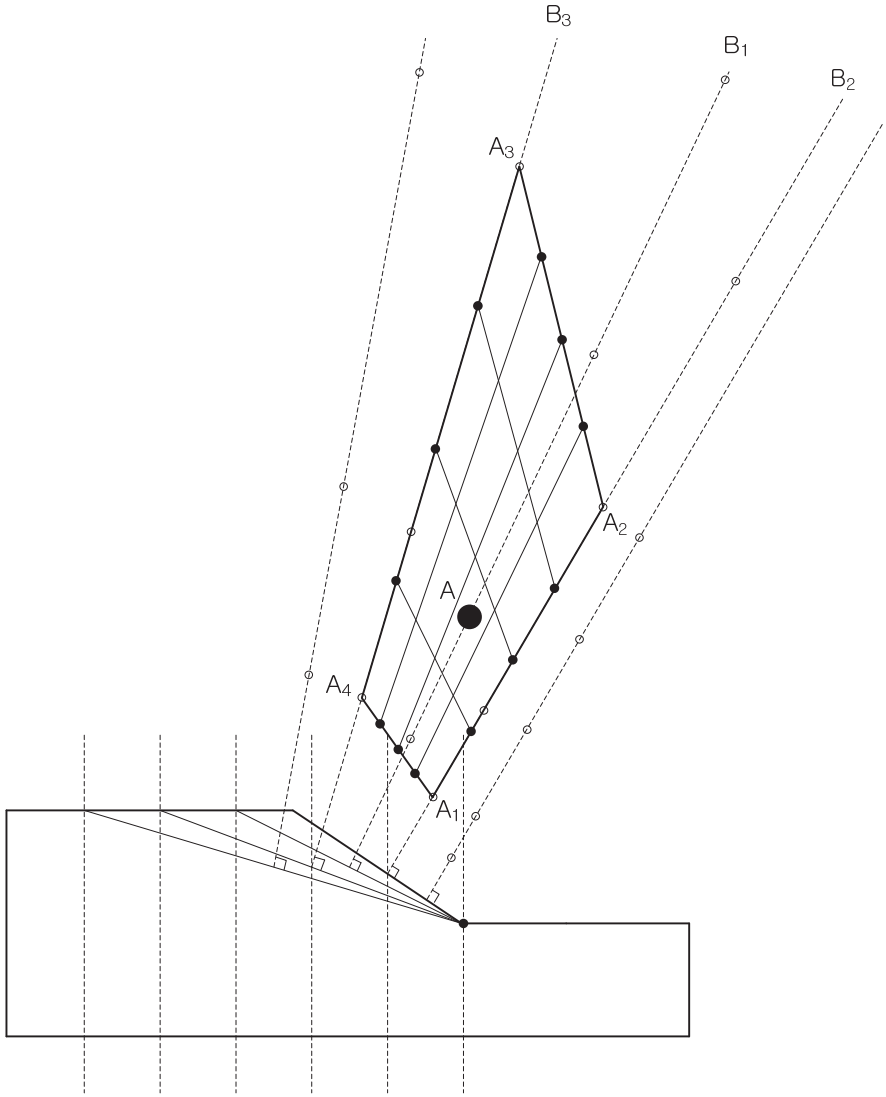


Figure 3.3.6 Auto-defining arc centers for the 2nd phase of analysis

3.3.3 Multi-Linear Failure Surface

Unlike defining arc failure surfaces, the zone of arc centers need not be defined for defining multi-linear failure surfaces. Each failure surface can be simply defined by multiple lines as shown in Figure 3.3.7.

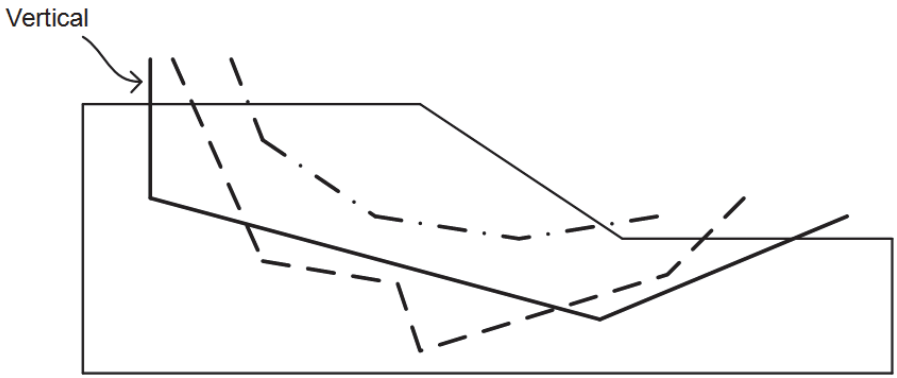
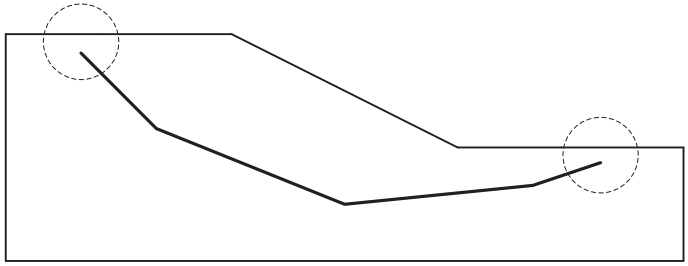
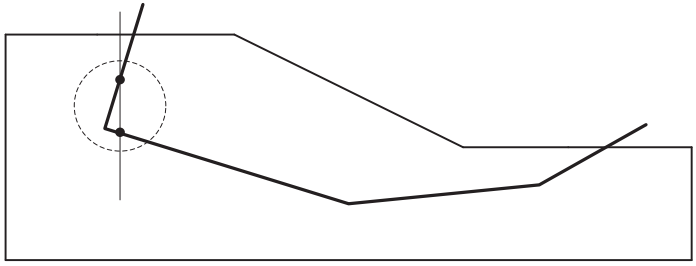


Figure 3.3.7 Examples of multi-linear failure surfaces

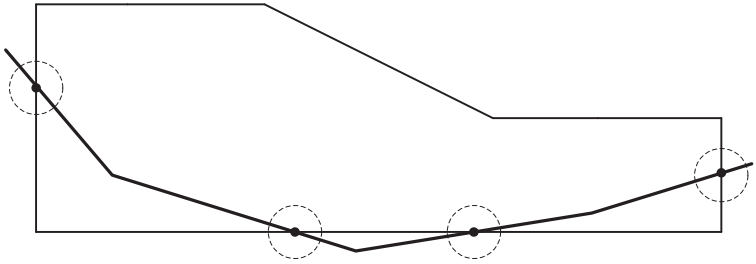
Figure 3.3.8 illustrates the cautionary points to be avoided when defining a multi-linear failure surface.



(a) Ends not meeting the slope surface



(b) Two points crossing a vertical line



(c) Failure surface crossing the outer extreme boundary faces

Figure 3.3.8 Shapes to be avoided when defining multi-linear failure surfaces

SoilWorks applies the three different methods to determine the safety factor that satisfies the equilibrium conditions when defining a multi-linear failure surface

(1) It may find the center of the circle close to the user-defined multi-linear failure surface using the least squared method and calculates the safety factor about the point.
(2) It may calculate the safety factor relative to an arbitrary reference point that user defined.
(3) The multi-linear failure surface is assumed to be a circle of which radius is infinite and it calculates the safety factor in the same manner for calculating the safety factor of the arc failure surface.

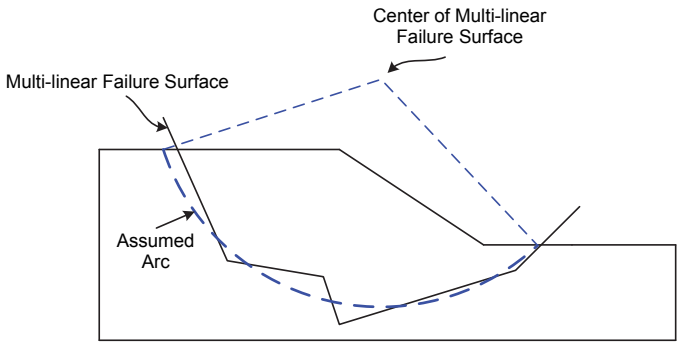


Figure 3.3.9 The center of the circle of the multi-linear failure surface

3.3.4 Cut Failure Surface

Cut Failure Surface is a function to find the minimum safety factor for the region corresponding to the intersection of the cutting line and the arc as shown in the figure below. In the field of practice, the failure surface does not occur as arc always by various conditions such as a soft layer. In such a case, the multi-linear failure surface shall be considered, but it is not suitable to consider the multiple cases of failure. Therefore, by using the arc failure surface and the Cut Failure Surface, the safety factor can be simply found.
The input method of the Cut Failure Surface is very similar to the one of the multi-linear failure surface explained before. However, because it is used as a secondary function of the arc failure surface, it does not take the input of the arc center point.

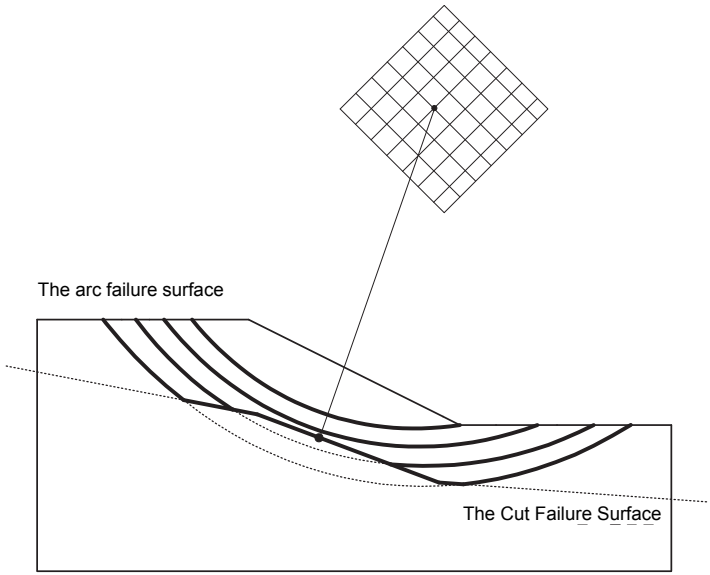


Figure 3.3.10 The arc failure surface and the Cut Failure Surface

3.3.5 Tension Crack

Tension Crack on the slope occurs frequently by osmosis of water. If the failure is in progress or can be expected by Tension Crack, this feature. Tension Crack is assumed to occur only in the vertical direction as shown below, and three types of input method are provided depending on preference.

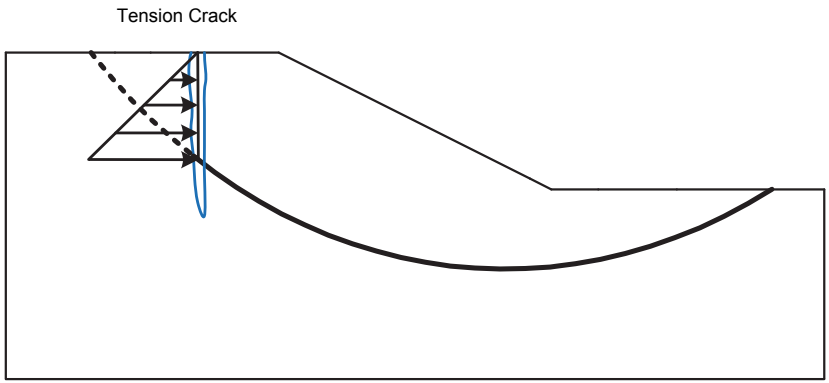


Figure 3.3.11 Tension Crack and water pressure

1. Auto Search: Once the normal force on the slice becomes negative, the afterward arc failure surface will be assumed not to have any resistance. After analysis, the slip surface extended up vertically from the base of the slice having negative normal force.

2. Tension Cracking Angle: If the failure surface is over a certain angle, it is assumed not to have any more resistance. Therefore, the arc failure surface will be cut vertically. The slip surface extended up vertically when the base angle of slice reaches defined angle. Note that the angle must be ranged from 45 to 65 degrees measured from the positive or negative x-axis.

3. Curve: if it is defined in the "Line", the tensile crack will occur at the intersection. The cutting will be cut after the line. With this option, the slip surface extended up vertically when the slip surface intersect the tension crack line.

Water may expand after it penetrates between tensile crack. In such a case, user may use the saturation and consider a certain pressure due to the expansion.

3.4 Slice Division

LEM is classified into the slice method, which must be used in the process of failure surface analysis investigated through arc failure surfaces or multi-linear failure surfaces.

The slicing method can be largely divided into the method of uniformly maintaining the slice width and the method of uniformly maintaining the length of the failure surface of each slice. The safety factors from the two methods are generally similar. However, if the slope of the failure surface is steep and/or reinforcement is included, some difference in the result may occur. Soilworks provides both methods, and engineers shall choose the method based on their engineering judgment.

3.4.1 Slice of Arc Failure Surface

An arc failure surface can be sliced at an equal length along the arc or at an equal width of slice.

In the method of the equal length, the length of the failure surface of each slice becomes L/n if the total length of the arc is L, and the number of divisions is n as shown in Figure 3.4.1.

If the shape of the arc failure surface is unrealistically defined as Figure 3.4.2, bending inward, the dotted line is ignored and replaced with the vertical line. In case of such vertical failure surface, stress is defined differently depending on the analysis method.

Fellenius method : $\sigma' = 0$

Bishop method : $\sigma' = \sigma_{\min}$

where,

If $\phi \neq 0$: $\sigma'_{\min} = -\dfrac{c}{\tan\phi}$, $\tau_{\max} = 0$

If $\phi = 0$: $\sigma'_{\min} = -\infty$, $\tau_{\max} = c$

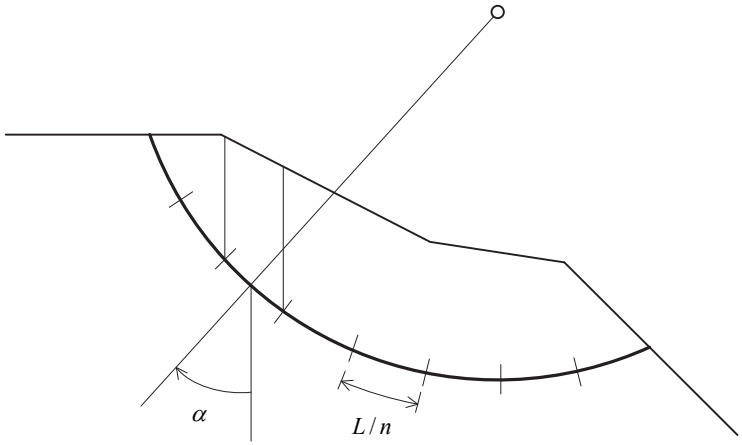


Figure 3.4.1 Method of dividing slices of arc failure surface (Equal Length)

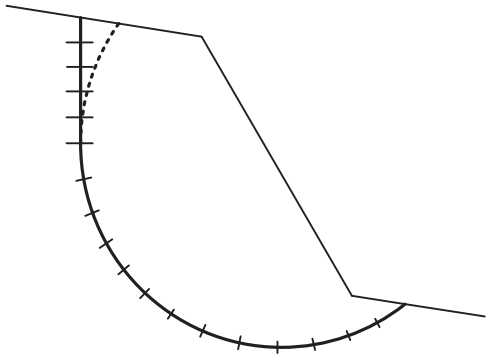


Figure 3.4.2 Defining vertical failure surface (Equal Length)

In the method of the equal width, the width of each slice becomes W / n if the width of arc is W, and the number of divisions is n as shown in Figure 3.4.3. In this method, the shape of the ground is taken into account when the arc failure surface is sliced.

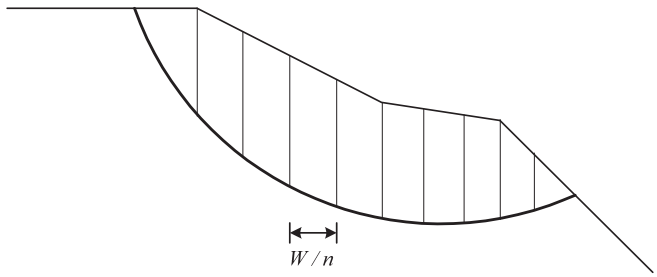


Figure 3.4.3 Method of dividing slices of arc failure surface (Equal Width)

If the arc goes above the center point, the slicing angle will be negative. In this case, the failure surface will be drawn vertically at the same height of the arc center point as shown in Figure 3.4.4

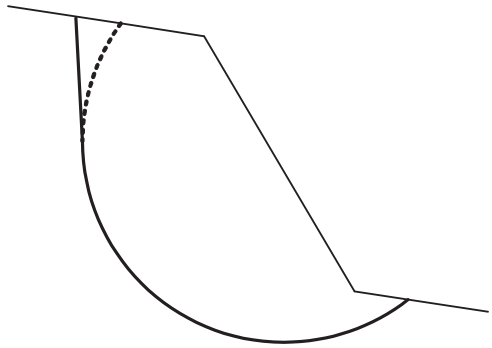


Figure 3.4.4 Defining vertical failure surface (Equal Width)

The failure surface of each divided slice is assumed to be a straight line as shown in Figure 3.4.5. The error stemming from the difference between the arc surface and the straight line is negligible and thus ignored.

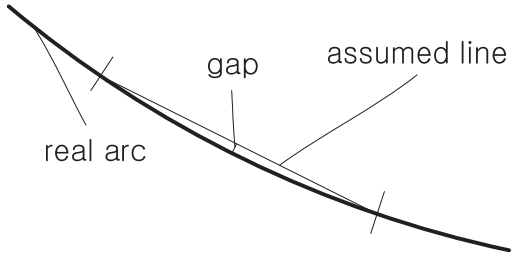


Figure 3.4.5 Error from the difference between the arc failure surface and the assumed straight line failure surface

3.4.2 Slice of Multi-Linear Failure Surface

In case of a multi-linear failure surface, each linear zone (i) is divided by the total division ratio as shown in Figure 3.4.4 and as follows:

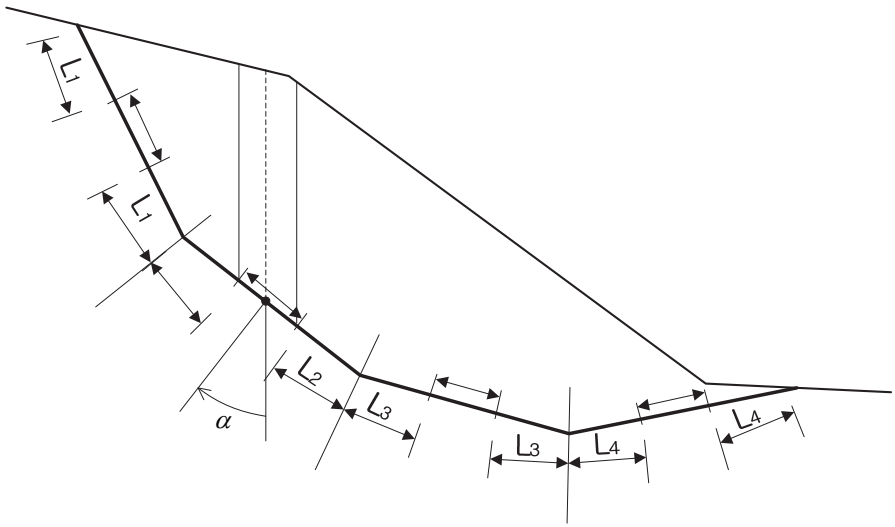


Figure 3.4.6 Division of multi-linear failure surface into slices

$$n_i = \frac{l_i}{\Sigma l_i} N$$
$$\Delta l_i = \frac{l_i}{n_i} \qquad (3.4.1)$$

where,

n_i	: Number of divisions in the straight line zone, i
l_i	: Length of the straight line zone, i
N	: Total number of divisions
Δl_i	: Number of divisions in the straight line zone, i

The equal width slicing option is added for the multi-linear failure surface as shown in Figure 2.4.7. Because the shape of the ground is considered when slicing, it may have more number of slices than dividing into equal length.

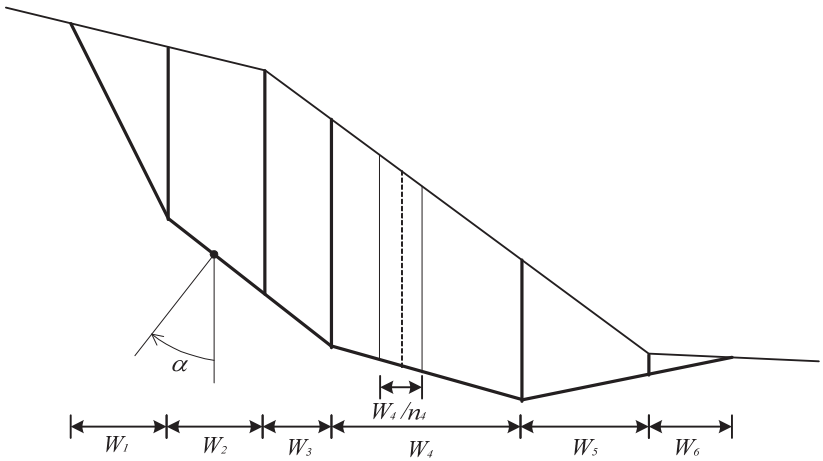


Figure 3.4.7 Division of multi-linear failure surface into slices (Equal Width)

3.5 Load Applications

3.5.1 Self Weight

SoilWorks automatically registers the geometry immediately upon modeling under the load set name, Self Weight, and assigns the self weight factor of -1.0 to the geometry in the global Z axis as shown in Figure 3.5.1. The value -1.0 signifies that the self weight load factor of 1.0 is applied to the unit weight, and the negative sign represents the gravity direction. For example if -1.5 is entered in the Z-axis entry field, then the unit weight is multiplied by the factor of 1.5 for calculating the self weight, which will act in the gravity direction. As the soil weight of a slope will always act in the gravity direction in LEM, the negative sign must be used at all times. Moreover, any value specified for the X-axis (horizontal) direction will be ignored due to the nature of LEM analysis.

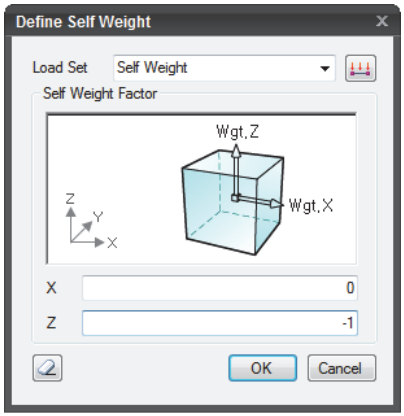


Figure 3.5.1 Self weight definition

If a water level is present as shown in Figure 3.5.2, the specified unit weight of the ground is used for the part A above the ground water level in the example slice. For the ground part B under the ground water level, the unit weight of the water is considered in addition to the unit weight of the ground in reflection of the concept of total stress. In order to accurately consider the weight of the part B, the saturated unit weight needs to be used, but the weight of water is simply added to the unit weight of the ground for the calculation of safety factors in reflection of the characteristics of simplified LEM analysis.

The submerged part C in Figure 3.5.2 is subjected to horizontal loading due to water pressure, which is accounted for in the calculation of safety factors.

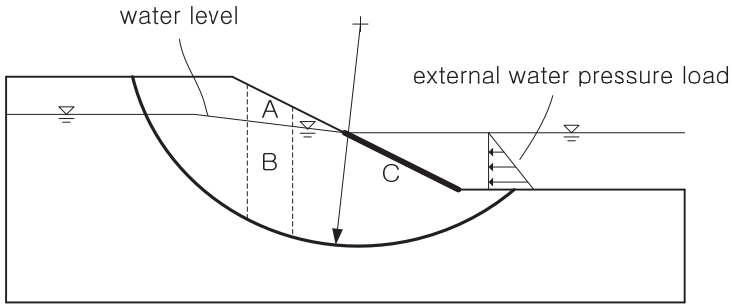


Figure 3.5.2 Reflection of self weight and water pressure

3.5.2 Line Load

In the LEM module of SoilWorks, static loads are classified into point load, line load and moment load.

3.5.2.1 Point Load

A point load in a 2-dimensional model represents a linearly distributed load in the thickness direction of a 3-dimensional model, which may be used to simulate train loads, self weight of a retaining wall, etc.

Point loads as shown in Figure 3.5.3 are considered in the calculation of safety factors only if the loads are acting within the sliding mass. The loads denoted in dotted lines in Figure 3.5.3 will be ignored.

Point loads can be specified in the GCS X and Z directions and the directions normal to the ground surface.

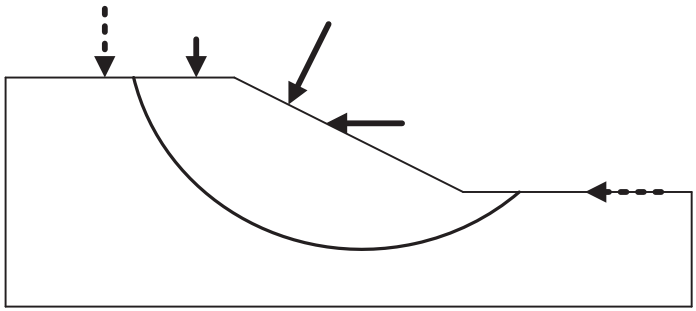


Figure 3.5.3 Definition of point loads

3.5.2.2 Line load

A line load in a 2-dimensional model represents a surface load distributed over a wide area in a 3-dimensional model. A vertically distributed load is applied to a slice and considered in the calculation of safety factors. A line load can be applied only in the gravity (GCS -Z) direction.

Line loads varying from the start to the end can be considered as shown in Figure 3.5.4 in the forms of trapezoidal and triangular distribution.

The linearly distributed loads indicated in dotted lines in Figure 3.5.4 are the loads specified by the user. Only the line loads applied to the sliding mass are searched and applied to the failure surface as shown in solid lines in Figure 3.5.4.

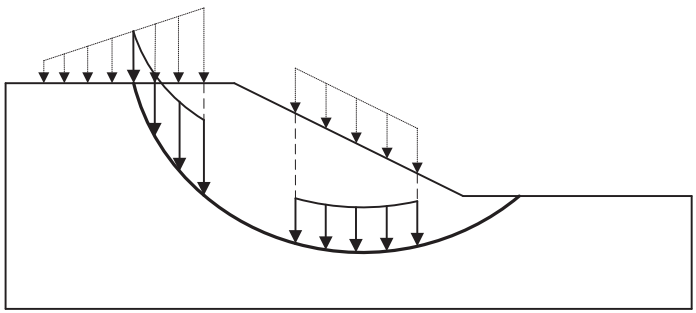


Figure 3.5.4 Applied line loads

3.5.2.3 Moment Load

A moment load is not widely used in practice, but can be used to represent a small scale retaining wall when a number of such retaining walls exist in a large slope.

The moment load indicated in a solid line within the sliding mass as shown in Figure 3.5.4 is considered in the calculation of safety factors. The dotted moment applied away from the sliding mass shown in the Figure will be ignored.

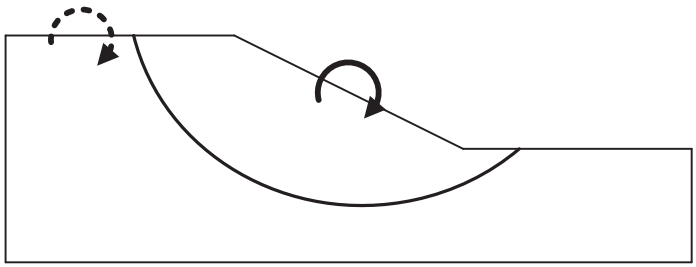


Figure 3.5.5 Applied moment loads

3.5.3 Static Earthquake Load

A static earthquake load represents a dynamic load generated during an earthquake, which has been transformed into a static load to be considered in LEM. Considering the characteristic of an earthquake load, it can be applied in both horizontal and vertical directions as shown in Figure 3.5.6. In the case of water contained within the ground, the earthquake factor acting on the water will be neglected.

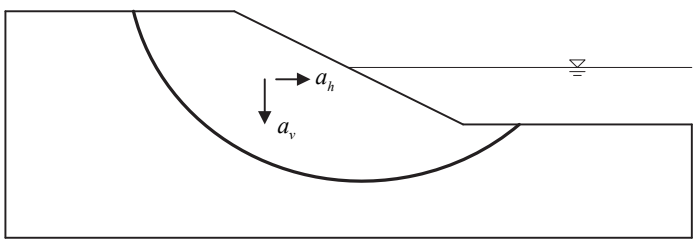


Figure 3.5.6 Use of earthquake factors

3.6. Definition of Ground Materials

In order to evaluate the capacity of the sliding surface of a slice in LEM, the principle of the Coulomb's friction is generally used. The friction principle defines the failure surface evaluated by the two parameters, cohesion (c) and internal friction angle (ϕ). In order to simulate realistic behaviors, the two parameters can vary depending on the depth, angle of sliding surface of a slice and the shape of $\sigma - \tau$ interaction.

3.6.1 Change in Cohesion due to Depth

The phenomenon of an increase in cohesion with depth is simulated to reflect the cohesion, c , at the sliding surface of each slice.

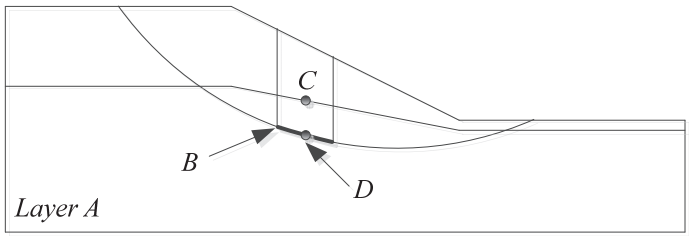


Figure 3.6.1 Change in cohesion with depth

$$c = c_0 + \Delta c\left(z_0 - z_i\right) \tag{3.6.1}$$

where,

c_0 : Initial cohesion at the layer A which includes the failure surface B of a slice

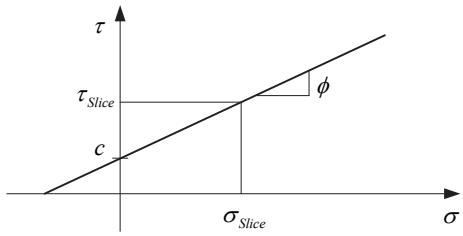
Δc : Incremental cohesion per unit depth (kN/m^3)

z_0 : Height of the top point C of the layer within which the failure surface of a slice exists

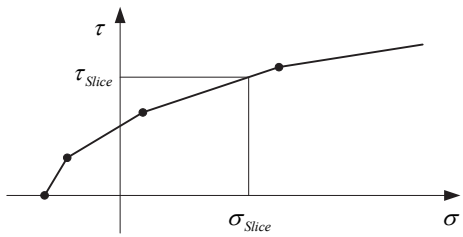
z_i : Height of the midpoint D of the failure surface B of a slice

3.6.2 Nonlinear Option

SoilWorks basically uses the Coulomb's failure criterion to define the resistance of the failure surface of a slice. But the Coulomb's failure criterion expresses the simple behavior of a linear type as shown in Figure 3.6.2(a), without being able to consider nonlinear type of behavior. In order to reflect more complex behavior of nonlinearity as shown in Figure 3.6.2(b), which may be obtained through material testing, SoilWorks supports a nonlinear option to account for the nonlinear nature of resistance at the failure surface of a slice.



(a) Coulomb's failure criterion



(b) User defined failure criterion

Figure 3.6.2 Failure criteria

3.6.3 Anisotropic Condition

In addition to the variation of cohesion with depth, cohesion also changes depending on the angle (α) of the failure surface of a slice as shown in Figure 3.6.3. SoilWorks enables the user to define such relationship. The angle (α) may be in the range between -90 and 90. The α angle is defined in Figure 3.4.1 (arrow direction is positive).

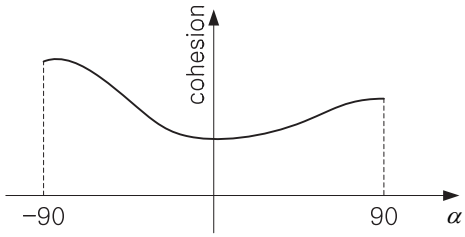


Figure 3.6.3 Change in cohesion due to the angle of failure surface in isotropic condition

When using the isotropic condition, “change in cohesion due to depth” or “nonlinear option” cannot be used.

3.7 Reinforcement

Slope failure generally occurs due to external factors such as ground cuts for roads and facilities, water seepage or external loading. Various reinforcement measures are known to exist in preventing such failure from occurring. The LEM module in SoilWorks supports the reinforcement of anchors, nails, strips and struts, the characteristics of which are outlined below for proper uses in the calculation of safety factors.

SoilWorks uses the diffusion width and angle of the reinforcement to realistically reflect the behaviors.

3.7.1 Anchor

Anchors can be used for reinforcing a slope against sliding. Anchors are prestressed and considered effective only in the axial direction, each of which is largely divided into the prestressing part and the anchorage part resisting the prestress.

The mechanism of an anchor resisting against sliding of a slope in the axial direction can be divided into the pull-out resistance, P_{Resist} and the tensile strength, T_{Yield}. The resistance of an anchor can be expressed as,

$$R_{Anch} = \min\left(P_{Resist}, T_{Yield}\right) \tag{3.7.1}$$

SoilWorks requires the tensile strength of an anchor in the input based on the smaller value between P_{Resist} and T_{Yield}.

The critical aspect of an anchor for prestressing is the anchorage. The bold lines in Figure 3.7.1 represent the anchorages, and the white dots represent the mid points of the anchorages. In the case of the anchor A, its anchorage exists in the ground beyond the failure surface in which case the calculation is based on the pull-out resistance, P_{Resist}. But in the cases of the anchors B and C, a special rule needs to be considered.

SoilWorks considers the anchor B (where the white dot is located beyond the sliding mass) as effective in resisting against slope sliding. In the case of the anchor C (where the white dot is located within the sliding mass), the anchor is considered ineffective and is thus

ignored in the calculation of safety factors.

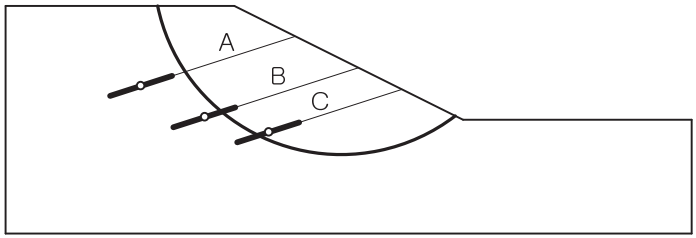


Figure 3.7.1 Effective range of anchors

In addition, the shear strength of the anchor is considered. It can be entered as a specific value or as a function. Using the function input, the function considers the change in the shear force according to the anchor's angle in respect to the horizontal direction as shown in Figure 3.7.2.

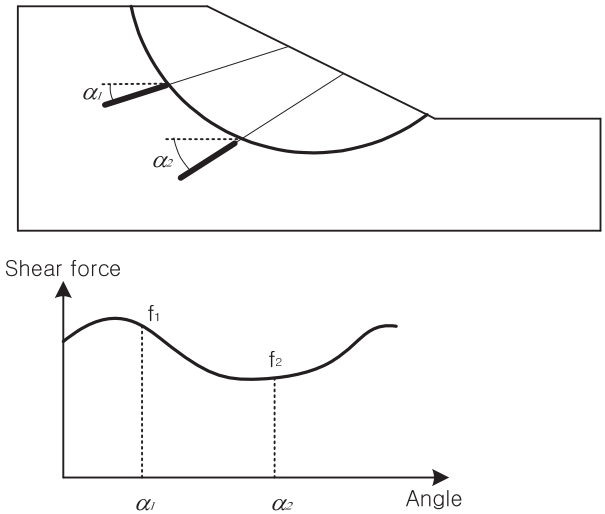


Figure 3.7.2 Shear Force Function

3.7.2 Nail/Pile

Nails are most widely used reinforcement materials to prevent slopes from sliding. Unlike anchors, nails are not prestressed, and the sections are relatively large. Because of the section sizes, the resisting mechanism against slope sliding can be divided into the axial resistance and shear resistance for which SoilWorks provides the following 3 options:

Table 3.7.1 Nail options provided in SoilWorks

Option	Axial direction	Shear direction
Nail	Value based on tensile strength and pull-out resistance	Constant value or input by a function
Pile	Not used	Calculated value of shear resistance
Nail + Pile	Value based on tensile strength and pull-out resistance	Calculated value of shear resistance

The input and calculation methods for the above table are explained below.

3.7.2.1 Definition of Axial Resistance

The axial resistance mechanism of a nail can be divided into the pull-out strength and the tensile strength as was done for an anchor in the previous section. The axial resistance can be also defined by Eq. (3.7.1) like an anchor. In the case of an anchor, the user specifies the smaller value of P_{Resist} and T_{Yield}. But in the case of a nail, the user specifies the tensile strength and the pull-out resistance per unit length, RCS, and the program automatically calculates the pull-out strength to determine the axial resistance, R_{Nail_axial} against sliding.

$$R_{Nail_Axial} = \min\left(P_{Resist}, T_{Yield}\right) \tag{3.7.2}$$

The pull-out resistance, P_{Resist}, is obtained by,

$$P_{Resist} = \int_0^{l_{ext}} \left(l_{ext} \times RCS\right) dl \tag{3.7.3}$$

where,

l_{ext} : Nail length beyond the failure surface (m)

RCS : Pull-out resistance per unit length (kN/m)

RCS can be calculated in two ways in SoilWorks. The user may directly enter the value of RCS, or the program can calculate RCS from the limit friction per unit area (q_S) of the ground surrounding the nail calculated as follows:

$$RCS = q_S \times \pi \times 2R \tag{3.7.4}$$

where,

q_S : Limit friction per unit area (obtained through CLOUTERRE, TA 86, Fascicule 62, testing or literatures)

π : Pi

R : Equivalent radius

The equivalent radius is generally based on the sectional area of a nail unless the nail cross section is increased due to grouting or cut-off of water.

If the option of only using the shear resistance is selected, the axial resistance is ignored and only the shear resistance below is considered.

3.7.2.2 Definition of Shear Resistance

1. Methods of defining Shear Resistance

Shear resistance can be defined by a constant value or a function, or can be calculated from the material properties.

The shear resistance of a nail, R_{Nail_shear}, is generally defined uniformly along the length of the nail, but the distribution of shear resistance can sometimes vary along the length as shown in Figure 3.7.3. SoilWorks supports both cases. The resistance distribution function is expressed in terms of shear values with respect to the distance from the nail head.

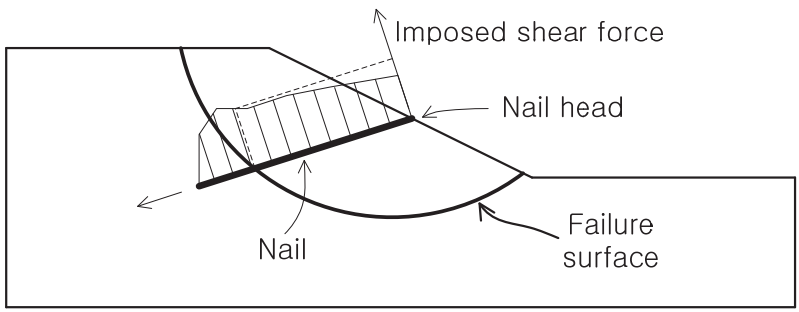


Figure 3.7.3 Distribution function of shear forces along the length of nail

Because no anchorage exists in a nail, contrary to an anchor, the nail is considered to resist slope failure even if a part of the nail crosses the failure surface.

2. Method of calculating Shear Resistance

Any reinforcement retaining the shear resistance property follows the Tresca's criterion. The shear resistance is not permitted to exceed 50% of the tensile strength.

$$T_{Axial} = 2T_{Shear} \tag{3.7.5}$$

$$\left(\frac{R_{Axial}}{2}\right)^2 + R_{Shear}{}^2 \le T_{Shear}{}^2 \tag{3.7.6}$$

where,

T_{Axial} : Tensile strength in the axial direction

T_{Shear} : Shear strength

R_{Axial} : Axial resistance

R_{Shear} : Shear resistance

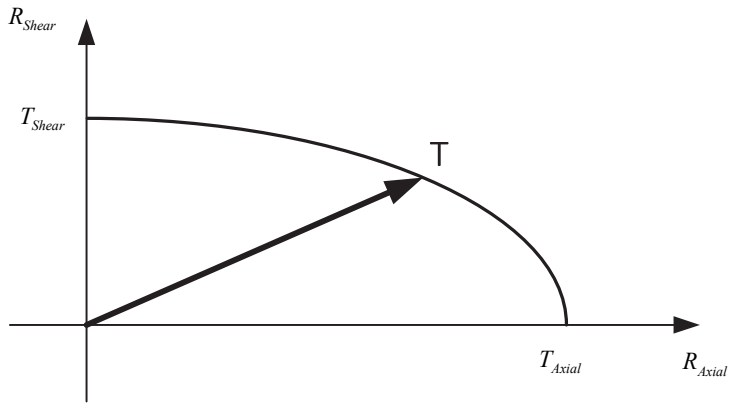


Figure 3.7.4 Safe zone of reinforcement resistance

Also, the shear resistance depends on the length of a nail due to its flexibility. The reference length is as follow:

$$L_{ref} = 2 \times \left(\frac{4EI}{E_{soil}}\right) \tag{3.7.7}$$

where,

EI : Flexural stiffness of reinforcement

E_{soil} : Modulus of elasticity in the transverse direction of ground

SoilWorks calculates the minimum shear resistance according to the reference length as

shown in the table below.

Table 3.7.2 Classification of nail reinforcement based on length

$L < L_{ref}$ (rigid)	$R_{shear1} = 0.25 \cdot p_{Limit} \cdot B \cdot L$ $R_{shear2} = 0.10 \cdot p_{Limit} \cdot B \cdot L + 4.05 \cdot M_{plastic} \left(1 - \frac{R_{Axial}^{2}}{T_{Axial}^{2}} \right) / L$
$L \geq L_{ref}$ (flexible)	$R_{shear1} = 0.25 \cdot p_{Limit} \cdot B \cdot L_{ref}$ $R_{shear2} = 0.12 \cdot p_{Limit} \cdot B \cdot L_{ref} + 3.24 \cdot M_{plastic} \left(1 - \frac{R_{Axial}^{2}}{T_{Axial}^{2}} \right) / L_{ref}$

where, p_{Limit} is the limit pressure of ground, B is the width of reinforcement, and $M_{plastic}$ is the plastic moment of the steel.

The maximum plastic moment of the reinforcement occurs at the point of maximum bending deformation as shown below.

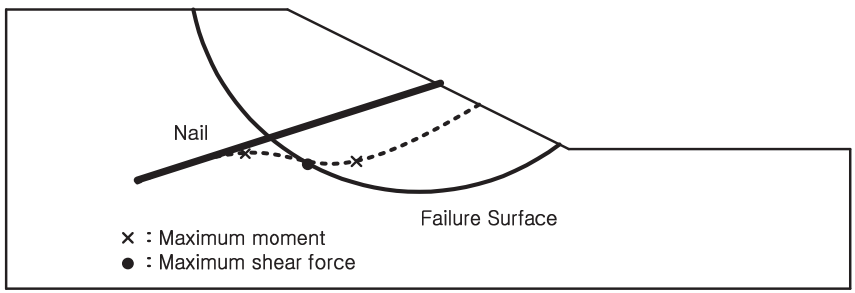


Figure 3.7.5 Location of maximum moment

Assuming that full plasticity has taken place at the point of maximum moment, the stress distribution then becomes,

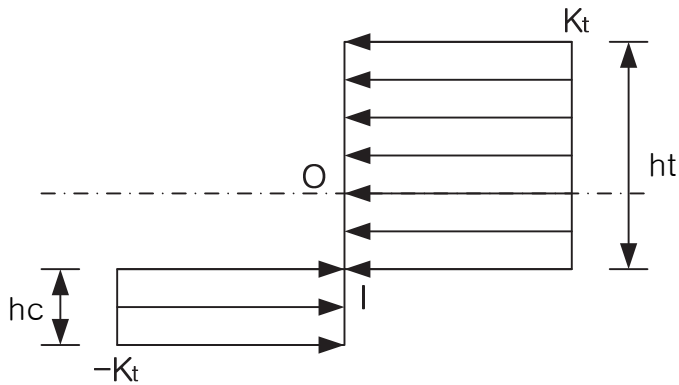


Figure 3.7.6 Shear stress at full plasticity

If a rectangular section with the width of b is assumed to have become fully plastic, the tensile force and the moment about I can be defined as follows:

$$T_{axial} = 2k_t b\left(h_t - h_c\right) \tag{3.7.8}$$

$$M_I = k_t b\left(h_t^2 + h_c^2\right) \tag{3.7.9}$$

where, k_t is the shear strength of the reinforcement, h_t is the tension zone, and h_c is the compression zone.

The maximum plastic moment is expressed as,

$$M_{plastic} = M_I - T_{axial}\left(\frac{h_t - h_c}{2}\right) \tag{3.7.10}$$

The maximum allowable moment of a simple beam is entered by the user, which is calculated as,

$$M_{allowable} = \frac{k_t b h^2}{2} \tag{3.7.11}$$

Therefore, the maximum plastic moment can be rearranged using the above equations as,

$$M_{plasitc} = M_{allowable}\left(1 - \frac{T_{axial}^{\ 2}}{R_{axial}^{\ 2}}\right) \tag{3.7.12}$$

Using the axial resistance and shear resistance calculated from Table 3.7.2, the diagram below can be expressed in which T represents the reinforcement force.

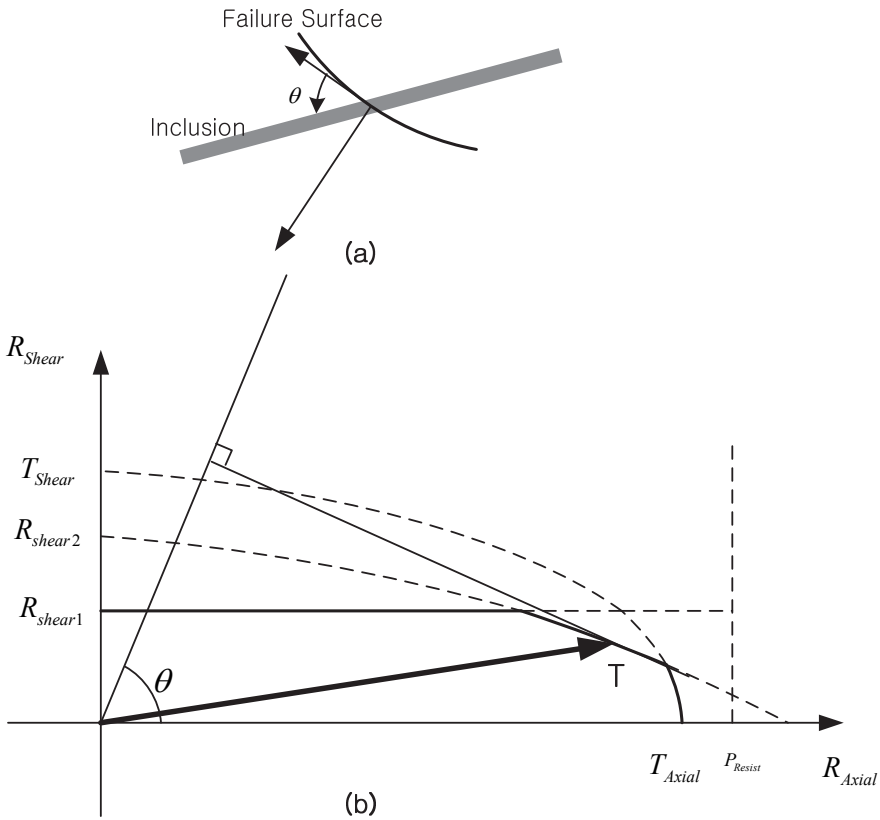


Figure 3.7.7 Diagram of calculating shear resistance

The reinforcement can be defined in 3 cases as shown in Figure 3.7.8 depending on the relative angle to the failure surface. Accordingly, the user needs to specify the critical

angle (θ_{crit}) additionally. At the boundaries (θ_{crit} , $\frac{\pi}{2} - \theta_{crit}$) of the critical angle, the factor of safety may drastically change for which the angle of less than 5° is generally defined.

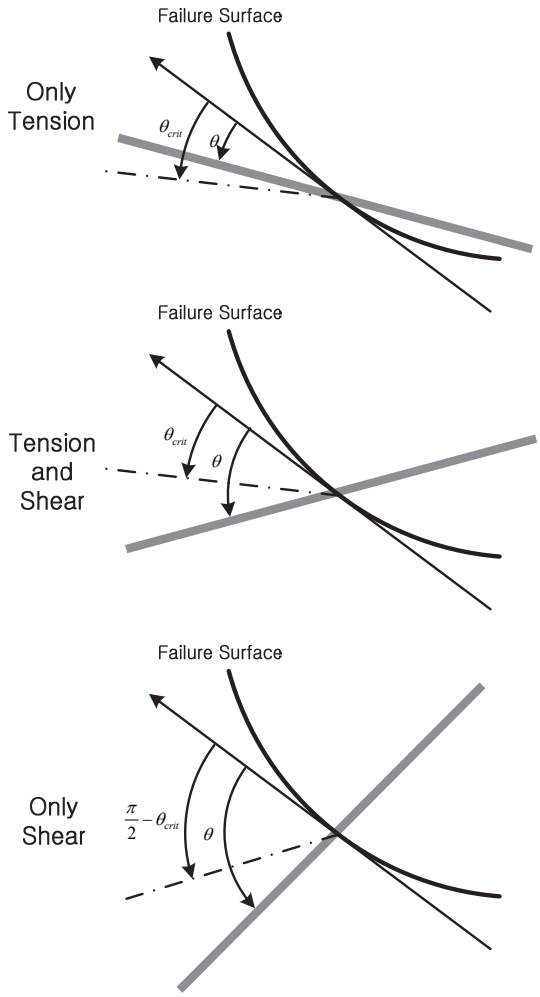


Figure 3.7.8 Use of critical angle

3.7.3 Strip & Fabric

Strip is a form of reinforcement made out of structural steel or fibers to resist slope failure. The mechanism of resistance mainly stems from the pull-out strength of the strip, which is generated by the friction force caused by the self-weight of the ground beyond the sliding mass following the Coulomb's Law. The maximum resistance $R_{strip/fabric}$ will be calculated as below.

$$R_{strip/fabric} = \int_0^{l_{ext}} i \cdot B \cdot \sigma_{v(input)} \mu dl \tag{3.7.13}$$

Here,

i : Number of friction faces

l_{ext} : Length of strip extending beyond the sliding mass

B : Width of strip

$\sigma_{v(input)}$: Vertical stress acting on the strip (User input)

μ : Coulomb's friction coefficient

Using detail method, the maximum resistance can be calculated using the vertical stress from the actual weight of ground used in the model.

$$R_{strip/fabric} = \int_0^{l_{ext}} i \cdot B \cdot \left(\tan\phi \cdot \sigma_{v(cal)} + c \right) dl \tag{3.7.14}$$

Here,

$\sigma_{v(cal)}$: Vertical stress acting on the strip (Calculated from the current model)

ϕ, c : Friction angle and cohesion

If l_{ext} is zero (i.e., strip only exists within the failure surface), the strip is ignored.

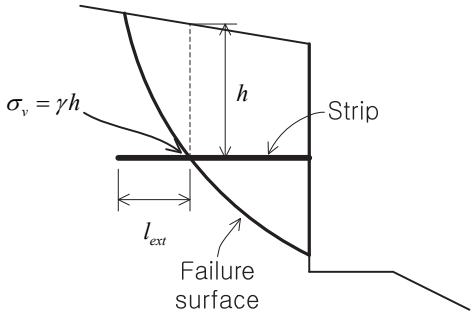


Figure 3.7.9 Strip resistance mechanism

3.7.4 Strut

Struts are installed to prevent the soil at an excavated face from flowing down, which are external structural support members. Although struts are applied as a type of reinforcements, struts are handled in a way similar to loads that act on the failure surface with a diffusion width and angle.

3.8 Calculation of Pore Water Pressure using Coupled Analysis

Pore water pressure is generally calculated through the seepage analysis of steady or transient flow. From this, it is not so easy to accurately determine the water level and apply it to the limit equilibrium method. The difficulty exists because general seepage analysis is performed by the finite element method, whereas LEM uses slices for analysis. SoilWorks has been designed to readily apply the pore water pressure results from seepage analysis to LEM through coupling the analysis results.

3.8.1 Determining Pore Water Pressure and Water Level at Failure Surface

In order to find the pore water pressure at a specific point, the element that encompasses the point is selected. The fact that the point generally does not coincide with a node of the element, the pore water pressure is calculated by interpolation of the nodal pore water pressures.

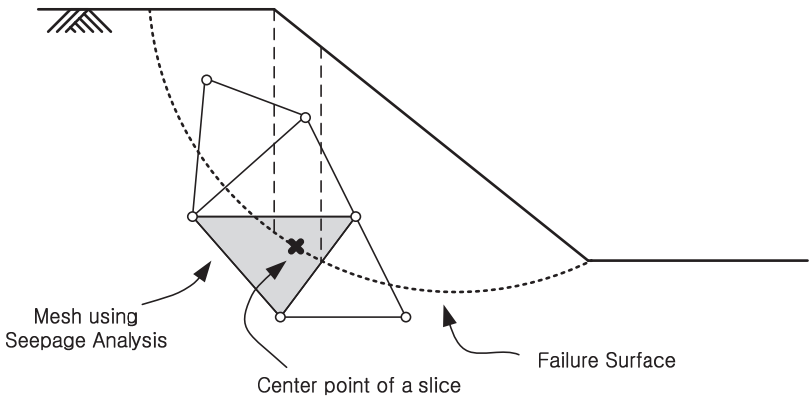


Figure 3.8.1 Calculation of pore water pressure at failure surface

SoilWorks uses the nodal pore water pressure and classifies the parts belonging to positive pore water pressure as the water level. Interpolation takes place in the area where both positive and negative pressures exist. Once the water level is determined, the saturated unit weight of soil is used under the water level and the dry unit weight is used above the water level.

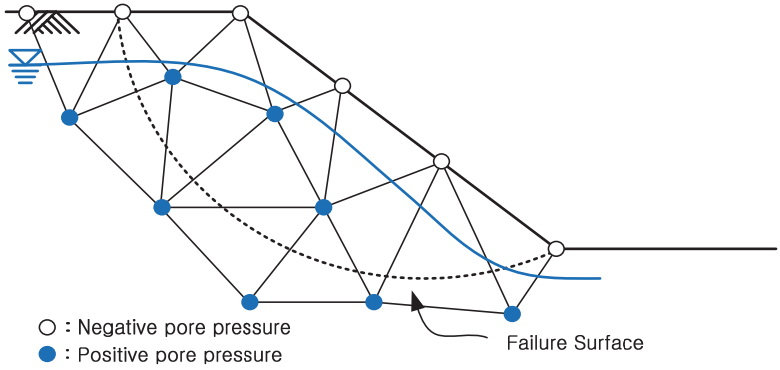


Figure 3.8.2 Determining water level using pore water pressure

Determining the pore water pressure through coupled analysis in LEM requires an iterative process of finding the corresponding elements at the points of interest. As such significant time may be consumed for searching in case there are too many points exist due to an excessive number of elements and slices. Accordingly, effective searching can be possible if the elements are divided into appropriate spacing as shown below, and only the elements within the corresponding bucket are searched.

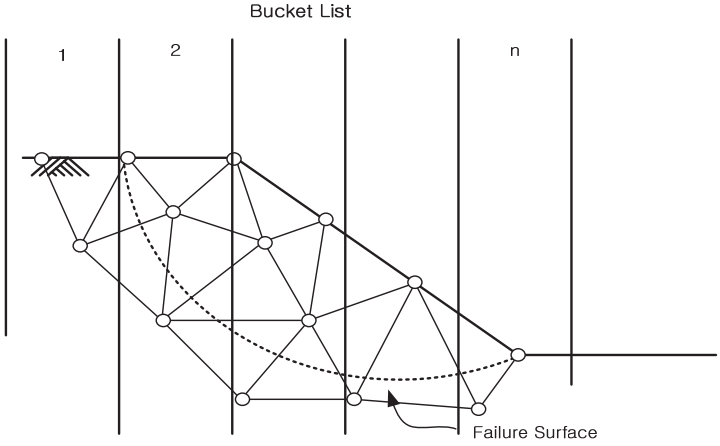


Figure 3.8.3 Division of elements using Bucket List

Chapter 4

Limit Equilibrium Method for Rock Slope

4.1 Introduction

The most important factors for analyzing and designing rock slope are the geological conditions of rock behind the slope, the direction of discontinuity surface and the continuity. Unlike earth, weathered soil and fractured soil, rock has discontinuity surfaces where unstable factors already exist. The geometrical properties of discontinuity surfaces and the mechanical properties due to shear strength (roughness, filling materials) are important.

In general, slope failure can be classified into the collapse such as rockslide and the sliding that occurs along the most vulnerable failure plane, based on its scale and behavior. Especially, rock slope failure is classified into 1) plane failure 2) wedge failure and 3) toppling failure according to the existing discontinuity surface and the combination of dip direction and dip of slope.

Like the arc failure mode in earth and weathered zone illustrated in Chapter 3, the limit equilibrium method is the most widely used for rock slope. As long as the scale of failure block is estimated accurately, the factor of safety can be calculated simply from the equilibrium condition of forces.

Figure 4.1.1 Failure modes of rock slope

The arc failure is identical to the failure mode of earth (weathered zone) slope, and the analysis can be performed using the limit equilibrium method of Slope Module. The plane failure and wedge failure, which are the typical rock failure modes, are separated into Rock Slope Module so as to model and analyze the failure block more conveniently. Stability analysis is performed first for the given rock slope model, and then the appropriate reinforcement is determined based on the failure mode.

4.2 Analysis Methods by Failure Modes

4.2.1 Plane Failure

Plane failure occurs when the dip of slope is greater than that of discontinuity surface, and it is a 2D analysis which does not take loads and forces into account in the depth direction as in plane strain elements. That is, the calculation is based on the following assumptions:

a) The strike of slip surface and tension crack plane is identical to that of slope and thus a 2D structure is formulated.
b) As all loads (weight, water pressure, reinforcement force, etc.) acting on the slip surface work with the center of gravity, the force equilibrium about rotation is always satisfied.
c) Both side faces of failure block are relaxation faces, and no resisting force acts on them.

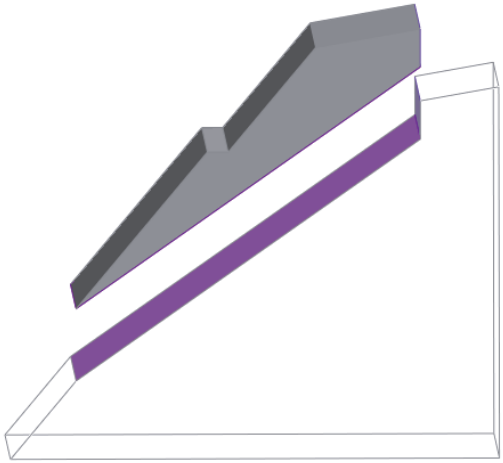


Figure 4.2.1 Geometrical shape of plane failure

4.2.2 Wedge Failure

Wedge failure occurs when the discontinuity surfaces in two different directions intersect. Therefore, the geometrical shape and the forces (driving force and resisting force) should be calculated in 3D, and not only the dip but also the rotational angle of dip direction should be taken into account. In addition, when the following kinematic conditions are met, the stability analysis for wedge failure can be performed:

a) The plunge of the intersection of two discontinuity surfaces should be greater than the internal friction angle of joint plane. That is, a failure occurs when the driving force that slides down is greater than the shear strength of joint plane.
b) The plunge of the intersection of two discontinuity surfaces should be smaller than the dip of slope. This is the geometrical condition that can form a failure block.
c) The trend of the intersection of two discontinuity surfaces should be parallel to the dip direction of the slope or intersect with the slope within ±20 deg.

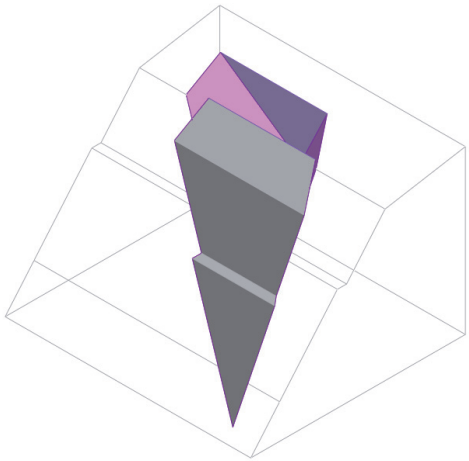


Figure 4.2.2 Geometrical shape of wedge failure

4.2.3 Stability Analysis

In Figure 4.2.3, the factor of safety is calculated by the force equilibrium when only the self weight is applied. Rock Module automatically considers the basic shape, variation in slope shape and various bench shapes when calculating the self weight of a failure block, which allows more accurate analysis than the existing method which calculates using an arbitrary average angle.

A variety of loads (static seismic load, water pressure, external load) and reinforcements (rock bolt, rock anchor) can be considered. Further details on these are discussed in separate sections.

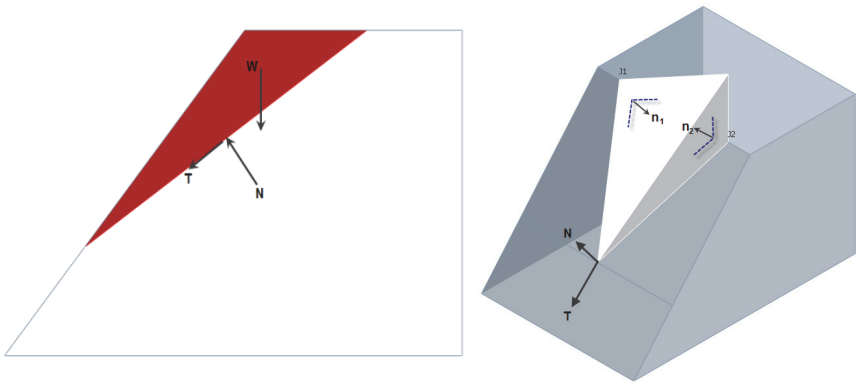


Figure 4.2.3 Forces due to self weight of failure block

The factor of safety (F.S) is expressed by the ratio of resisting force to driving force as below:

$$\mathbf{F.S.} = \frac{\textbf{Resisting Force}}{\textbf{Driving Force}}$$

If the self weight of failure block is considered only and the strength characteristic of joint plane satisfies the Mohr-Coulomb failure criterion, the factor of safety is defined as:

$$\mathbf{F.S.} = \frac{\tau \times l_{fail}}{T} = \frac{(\sigma_n \times \tan\emptyset + c) \times l_{fail}}{W \times \sin\theta}, \ \sigma_n = N \times l_{fail}$$

where, W is the self weight of failure block, N is the normal force due to self weight, T is the driving force due to self weight, σ_n is normal stress, τ is shear strength, l_{fail} is the

length of failure plane, ϕ is friction angle, c is cohesion, and θ is failure angle. In case of wedge failure, normal force (N) due to self weight exists separately on the failure plane and can be calculated by the following equation:

$$\mathbf{N_1} = \frac{(\mathbf{W} \times \mathbf{n_2}) \cdot (\mathbf{n_1} \times \mathbf{n_2})}{(\mathbf{n_1} \times \mathbf{n_2}) \cdot (\mathbf{n_1} \times \mathbf{n_2})}, \mathbf{N_2} = \frac{(\mathbf{W} \times \mathbf{n_1}) \cdot (\mathbf{n_2} \times \mathbf{n_1})}{(\mathbf{n_2} \times \mathbf{n_1}) \cdot (\mathbf{n_2} \times \mathbf{n_1})}$$

If additional loads other than the self weight, such as water pressure, static seismic load, external load, reinforcement force, etc, are considered, they will increase the driving force or decrease the resisting force. When all these loads act on the failure block at the same time, the above general equation can be solved as shown in the below example. All forces are applied to the center of gravity of the failure block, and therefore, the moment equilibrium is always satisfied.

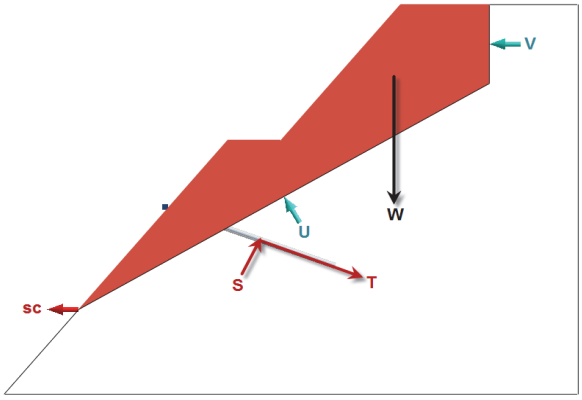


Figure 4.2.4 Forces due to plane failure

$$\mathbf{F.S.} = \frac{\mathbf{cA} + (\mathbf{W}(\mathbf{cos\beta} - \mathbf{sc} * \mathbf{sin\beta}) - \mathbf{U} - \mathbf{Vsin\beta} + \mathbf{Tcos\theta} - \mathbf{Ssin\theta})\mathbf{tan\emptyset}}{\mathbf{W}(\mathbf{sin\beta} + \mathbf{sc} * \mathbf{cos\beta}) + \mathbf{Vcos\beta} - \mathbf{Tsin\theta} - \mathbf{Scos\theta}}$$

where c and Φ are the cohesion and friction angle of joint plane, A is the length of joint plane, W is the weight of failure block, sc is seismic coefficient, U is the water pressure acting on joint plane, V is the water pressure acting on tension crack plane, T is the tensile force of reinforcement, S is the shear force of reinforcement, θ is the angle between reinforcement and joint plane, and β is the dip of joint plane where a failure is probable.

4.3 Material Properties of Soil

4.3.1 Soil Material Definition

The most important factors in calculating the factor of safety of rock slope are the exact estimation of the scale of failure block and the definition of shear strength of the joint plane that slides. Most commonly used five material models are available so as to be able to define the strength characteristic of joint plane according to the types of site condition and measured or experimental data. In case of wedge failure, different models can be selected for the two faces.

(1) Mohr-Coulomb

Mohr-Coulomb is the most typical material model of soil, and the relationship between shear strength and normal stress can be expressed as:

$$\tau = \sigma_n \cdot \tan\phi + c$$

where, τ is shear strength, σ_n is normal stress, ϕ is friction angle and c is cohesion.

(2) Barton-Bandis

The shear stress by Barton-Bandis is defined as follows:

$$\tau = \sigma_n \cdot \tan\left[\phi_b + JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

where, ϕ_b is the basic friction angle of failure plane, JRC is the roughness coefficient of contact, and JCS is the compressive strength of contact.

(3) Hoek-Brown

The limit strength by Hoek-Brown is as follows:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m\frac{\sigma_3}{\sigma_{ci}} + s\right)^{1/2}$$

where, σ_1 is maximum principal stress, σ_3 is minimum principal stress, σ_{ci} is the uniaxial compressive strength of rock, and m and s are the material constants of rock.

(4) Generalized Hoek Brown

The generalized Hoek Brown equation is as follows:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a}$$

(5) Power Curve

The shear strength by Power Curve is expressed as:

$$\tau = a\left(\sigma_n\right)^b + c$$

where, τ is shear strength, σ_n is normal stress, c is cohesion, and a and b are the material constants of rock.

4.3.2 Roughness and Filling Materials

It is difficult to accurately define the mechanical behavior of rock joint plane with one material model. To define the site condition more precisely and reflect it in the analysis, Rock Module provides the additional options as below. Filling materials affect the factor of safety based on the type of filling material and filling ratio, irrespective of the shear strength characteristic of joint plane.

(1) Roughness

As the roughness of joint plane increases, the shear strength increases and the factor of safety also increases linearly.

As shown in the following equation, the shear strength is added as much as the roughness angle to the normal stress.

$$\tau_R = \sigma_n \cdot \tan\left(\phi_R\right)$$

where, τ_R is the shear strength of roughness, and ϕ_R is the friction angle of roughness.

(2) Filling Materials

Filling material is one of the main factors that governs the shear behavior of joint plane besides the strength, roughness and water pressure, and fills the voids between the joint planes. Most of the filling materials are sand, silt and clay, which reduce the shear strength of joint plane. The thickness and the strength characteristic of filling material decide the stability of rock slope. When the filling materials are taken into account, the shear strength of joint plane is defined as:

$$\mu = \mu_{\max} - \left(\mu_{\max} - \mu_{\min}\right)\tanh\left(m\frac{t}{a}\right)$$

where, μ is the stress ratio ($=\tau/\sigma_n$), $\mu_{\max}$ is the stress ratio of unfilled joint plane, $\mu_{\min}$ is the stress ratio of filling material, m is material constant, and $\frac{t}{a}$ is the ratio of filling thickness to asperity height.

4.4 Loading

4.4.1 Water Pressure

Unlike earth slope, as the rock itself has substantially low permeability, the effect by the pressure load of the infiltrating water between joint plane and tension crack plane is considered. The water pressure (u) can be defined by the following equation, according to the distribution type and water level (percent filled %).

$$u = \int_0^{hw} \gamma_w \cdot h \cdot \xi(h) \cdot t \cdot dl$$

where, γ_w is the unit weight of water, h is the depth from water level, $\xi(h)$ is the shape factor (SoilWorks provides linear, triangular and uniform distributions), and t is the thickness in the depth direction.

The calculated water pressure decreases the resisting force and increases the driving force against the self weight of failure block in the normal direction of each discontinuity surface, and is the most critical factor that impedes the slope stability.

Water pressure has different distribution types depending on the permeability of filling material; that is, the presence of filling material, the drainage properties of slope bottom. Rock Module provides the following distribution types of water pressure depending on the tension crack:

(1) Linear Distribution

The maximum water pressure is applied to the starting point of joint plane. Among four distribution types, linear distribution has the largest maximum water pressure. This is applicable when the groundwater does not flow out of the starting point of joint or the slope bottom because there are clay filling materials of low permeability or freezing.

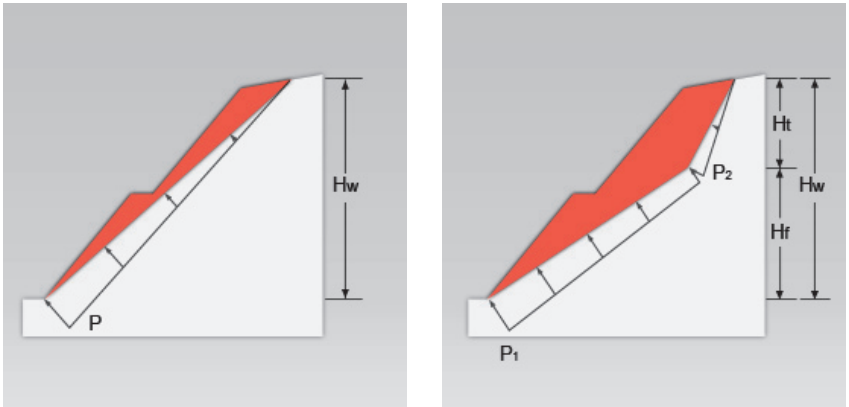


Figure 4.4.1 Linear distribution

(2) Triangular Distribution

The maximum water pressure is applied to the middle point. Water pressure is zero at the starting point and the end point of joint because it linearly increases and decreases. This is applicable when the groundwater is flowed because the slope bottom is well-drained.

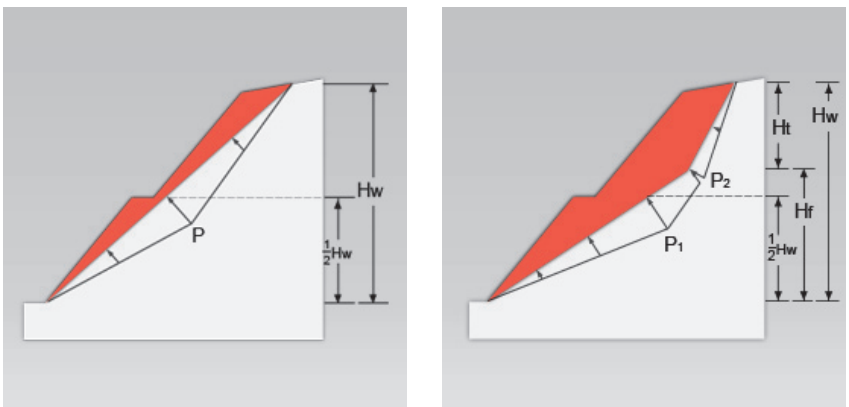


Figure 4.4.2 Triangular distribution

(3) Uniform Distribution (User Defined)

When the actual data of water pressure is available, this option is useful. User can use his/her discretion. Average water pressure is manually entered to joint plane and tension crack plane as pressure loads. It is not necessary to enter the unit weight of water and the percent filled (groundwater level), which are used to automatically calculate the water pressure.

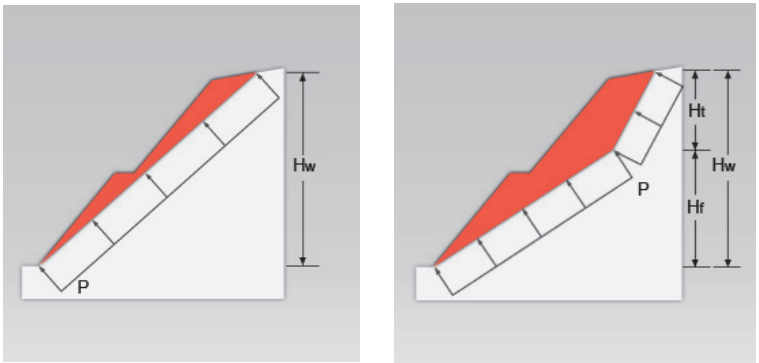


Figure 4.4.3 Uniform distribution (user defined)

(4) Tension Crack Criteria

This is applicable when the groundwater level exists along tension crack and the slope bottom is well-drained as in the triangular distribution. When the heavy rain is concentrated for short-term or the clay filling materials which have low permeability exist on slip surface (joint plane), water pressure acts only on the tension crack and is less likely to act on the slip surface. In this case, water pressure can be excluded from the joint plane.

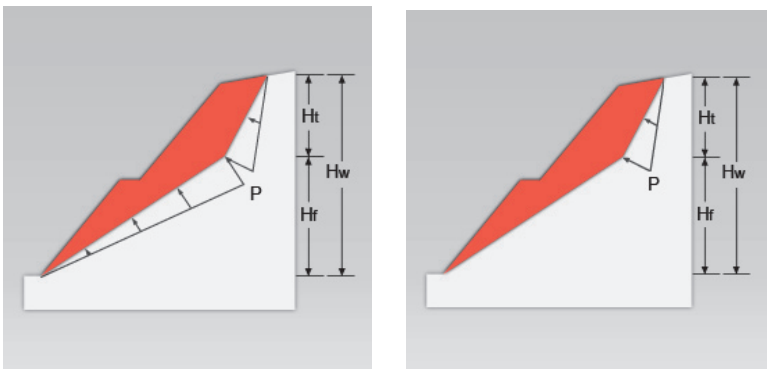


Figure 4.4.4 Tension crack criteria

4.4.2 Static Seismic Load

Static seismic load refers to the static load converted from the dynamic load that occurs during earthquake.
It is possible to directly enter the load with respect to the global coordinate system as well as the horizontal direction/joint plane direction, depending on the user's intention. In general, the failure block weight multiplied by 50% of maximum horizontal ground acceleration of seismic wave is converted to an equivalent static load and then the stability is checked. If the maximum horizontal ground acceleration due to seismic load is a_{max}, the lateral load that is additionally applied to the failure block is calculated as follows:

$$K_h = a_{max} / g \ (\times 50\%) \ W$$

where, a_{max} = maximum horizontal ground acceleration, W = the weight of failure block, and g = gravity acceleration

The seismic coefficient is applied by multiplying the importance factor, which reflects the seismic zone factor and the seismic capacity category by soil types as shown in the following table:

Soil type	Seismic zone	Seismic category		
		First class	Class 1	Class 2
SA	I	0.18	0.13	0.09
	II	0.10	0.07	0.05
SB	I	0.22	0.15	0.11
	II	0.14	0.10	0.07
SC	I	0.26	0.18	0.13
	II	0.16	0.11	0.08
SD	I	0.32	0.22	0.16
	II	0.22	0.15	0.11
SE	I	0.44	0.31	0.22
	II	0.34	0.24	0.17

4.4.3 External Load

External load can be used to apply a structural load to the slope top or to increase the resistance in the starting point of joint/bench. Loads can be applied in multiple directions and magnitudes at the same time. The applied force is considered as an axial load with reference to the entered angle, like the tensile force of reinforcement.

4.5 Reinforcement

Rock Module provides two types of reinforcements, which are Rock Anchor and Rock Bolt. Tensile force is compared with pull-out force and the minimum is taken, as shown in the below formula. Shear force is not considered for Rock Anchor.

$$R_{axial} = \min\left(P_{tension},\ P_{pullout}\right)$$

$$P_{pullout} = \int^{l_{ext}} q \cdot \pi \cdot D\ dl$$

where, R_{axial} is the axial force of reinforcement, $P_{tension}$ is tensile force, $P_{pullout}$ is pull-out force, q is the pull-out stress per unit area, D is the diameter of reinforcement, and l_{ext} is the external length of failure surface of reinforcement.

Rock Module automatically calculates the external length of failure surface of individual reinforcement (grouted length) after placement, and calculates the pull-out force.

Appendix

A. Definition of Diffusion Width and Diffusion Angle

When reinforcing members are installed through a failure surface, the true effect of the reinforcement propagates into the surrounding zone beyond the actual place of installation. In order to account for such distribution effect of the reinforcement, SoilWorks adopts the concept of diffusion width and angle. The diffusion zone is depicted in Figure A.1.

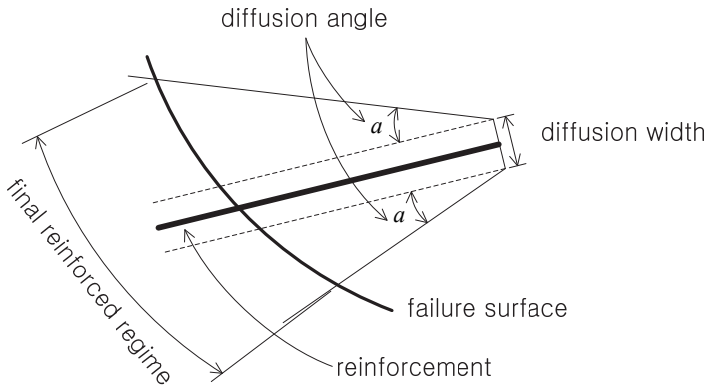


Figure A.1 Diffusion width and angle of effective reinforcement

Apart from the fact that the distribution effect exists and should be reflected, appropriate magnitudes of the diffusion width and angle are not so well known. Generally, the average width of the reinforcement head is used for the diffusion width, and the diffusion angle should be based on experimental data in the absence of which 10 – 20 degrees may be used to define the diffusion angle.

B. Distribution Ratio of Reinforcement

The effect of reinforcement based on the diffusion width and angle is outlined in Appendix A. At this point, the effect of the distributed reinforcement force in a slice is greater if the angle with the reinforcement is small. Conversely, the effect is smaller if the angle is large. In order to reflect such distribution effect, the reinforcement force calculated in Section 3.7 is multiplied by the distribution ratio, ρ calculated below to calculate the distributed reinforcement force in each slice.

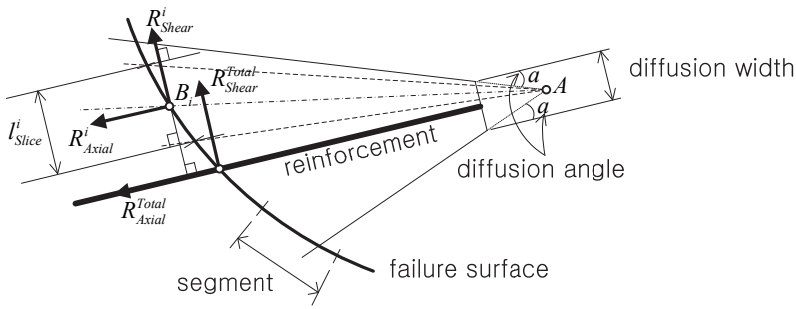


Figure B.1 Diagram of reinforcement force distributed in i th slice

First, the distance from the point A defined by the diffusion width and angle in Figure B.1 to the center of the i th slice, B_i is calculated. And after a line is drawn to the point B_i perpendicular to the reinforcement, the length (l^i_{Slice}) projected by a slice onto the perpendicular line is calculated. Such calculation for all the slices is thus completed, which is used to determine the distribution ratios, which are in turn used to calculate the slice force of the i th slice as below.

$$R^i_{Axial} = \rho_i R^{Tot}_{Axial} \tag{B.1a}$$

$$R^i_{Shear} = \rho_i R^{Tot}_{Shear} \tag{B,1b}$$

ρ_i represents the distribution ratio of the reinforcement force at the i th slice, which is calculated using l^i_{Slice} and $\overline{AB_i}$ as follows:

$$\rho_i = \frac{\dfrac{l^i_{Slice}}{\overline{AB}_i}}{\sum_i \dfrac{l^i_{Slice}}{\overline{AB}_i}} \tag{B.2}$$

In addition, the following equations are established to satisfy the force equilibrium:

$$R^{Total}_{Axial} = \sum_i R^i_{Axial} \tag{B.3a}$$

$$R^{Total}_{Shear} = \sum_i R^i_{Shear} \tag{B.3b}$$

Eq. (B.3) however does not satisfy the moment equilibrium.

Based on Eq. (B.1) and Eq. (B.3), the distribution ratios of slices must satisfy the following condition:

$$\sum_i \rho_i = 1 \tag{B.4}$$

If the diffusion angle, a is assigned 0 (zero), the point A does not exist in which case the distribution ratios are determined as,

$$\rho_i = \frac{l^i_{Slice}}{\sum_i l^i_{Slice}} \tag{B.5}$$

The reinforcement force distributed to the i th slice through Eq. (B.1) acts in the reinforcement direction, which is then transformed into the actual slice forces, R^i_{Normal} and $R^i_{Tangential}$ normal and tangential to the failure surface as shown in Figure B.2.

$$R^i_{Normal} = R^i_{Axial}\sin(\alpha_i + \beta) - R^i_{Shear}\cos(\alpha_i + \beta) \tag{B.1a}$$

$$R^i_{Tangential} = R^i_{Axial}\cos(\alpha_i + \beta) + R^i_{Shear}\sin(\alpha_i + \beta) \tag{B.1b}$$

where,

β : Angle between the reinforcement and normal direction)

α_i : α angle of the i th slice as shown in Figure 3.4.1

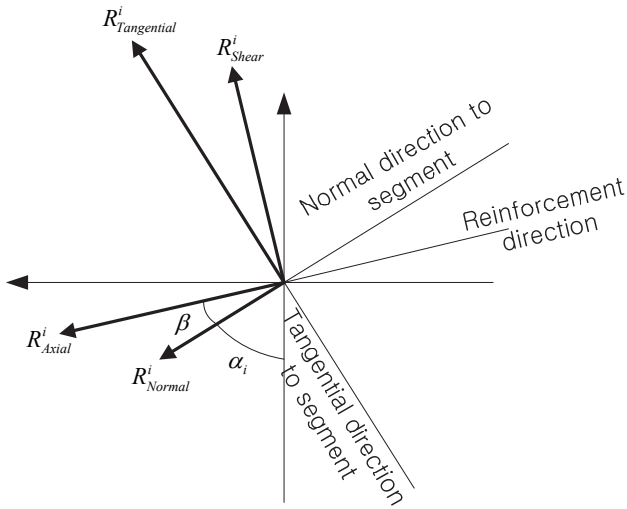


Figure B.2 Definition of β